IMAGE RESTORATION USING A HYBRID COMBINATION OF PARTICLE FILTERING AND WAVELET DENOISING

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ABSTRACT

In this paper we propose a novel image restoration method that effectively combines a particle filter with wavelet shrinkage to achieve robust performance against inhomogeneous noise mixtures. Specifically, the particle filter acts to suppress outlier-rich components of the noise while, in a subsequent step, the wavelet domain shrinkage attenuates any remaining, less heavily tailed noise components. We present late breaking preliminary examples demonstrating excellent rejection of salt-and-pepper like Cauchy noise mixed with additive white Gaussian noise (AWGN). Although limited in scope, these preliminary results suggest that the combination of particle filters with more traditional restoration techniques is a powerful approach that can provide a new dimension of flexibility for addressing noise mixtures involving difficult nonlinear and non-Gaussian components.

1. INTRODUCTION

The objective of restoration is to recover an image that has been corrupted by distortion and noise. In general, the distortion could be linear or nonlinear and the noise might be Gaussian, heavily tailed, or both [1]. Traditional approaches typically involve a tradeoff between a deconvolution process designed to combat the distortion and a denoising process. In this paper, we focus on the denoising part of the problem and specifically consider noise mixtures involving simultaneous heavily tailed and Gaussian components. Although the median filter is ideal for rejecting impulsive noise, its performance has been shown to be suboptimal when Gaussian noise components are also present [2].

Shrinkage methods based on thresholding in the discrete wavelet domain have attracted significant attention in image denoising applications [3]- [5]. The basic idea is to decompose the noisy image using the discrete wavelet transform (DWT) and then apply thresholding, where it is assumed that small coefficients in the high frequency (small spatial scale) subbands are associated with noise and can be set to zero without substantially affecting the main visually important features of the image.

In this paper we consider the combination of particle filtering (PF) with wavelet shrinkage as a new hybrid approach for dealing with difficult, inhomogeneous noise mixtures containing both Gaussian and heavily tailed components. The PF technique has emerged as a superior method for treating nonlinear and non-Gaussian problems [6]- [8] and has been applied recently for image and video processing [9]- [11]. Also known as sequential importance sampling (SIS), PF is a Monte Carlo (MC) method that propagates a set of weighted samples (or *particles*) to simulate the posterior distribution of a system's trajectory in state space. Since PF inherently supports nonlinear dynamics and non-Gaussian forcing and measurement noises, we use it here to suppress the heavily tailed components of the corrupting noise, whereas wavelet shrinkage is applied in a subsequent step to attenuate the Gaussian noise components.

2. PROPOSED METHOD: PF-DWT DENOISING

When an image is corrupted by noise comprising both heavily tailed and Gaussian components, we have observed that undesirable blurring artifacts are often introduced if one attempts to suppress both components simultaneously. Therefore, we introduce a two-stage hybrid denoising technique called *PF-DWT*. In the first stage, a spatial PF is used to suppress the heavy tailed component of the noise. In the second stage, wavelet thresholding is applied to attenuate the remaining "Gaussian-like" noise. The input of the second stage depends on the output of the first stage, thus realizing a synergistic hybrid approach that is robust and effective against a wide variety of inhomogeneous noise mixtures.

2.1. General Form of The Particle Filter

In this section, we briefly review the general PF framework described in [6] - [8]. Consider a nonlinear system modeled in state space according to

$$\boldsymbol{x}_t = f(\boldsymbol{x}_{t-1}) + \boldsymbol{v}_{t-1}, \quad (1)$$

$$\boldsymbol{y}_t = h(\boldsymbol{x}_t) + \boldsymbol{n}_t, \quad (2)$$

where $\boldsymbol{x}_t$ and $\boldsymbol{y}_t$ denote the hidden states and the observations of the system at time t, respectively. Both $f(\cdot)$ and $h(\cdot)$ could be nonlinear functions. For image processing, the time index t is replaced by spatial indices i and j representing the row and column of a pixel, respectively. The process and observation noises, which are assumed non-Gaussian, are given by $\boldsymbol{v}_t$ and $\boldsymbol{n}_t$, respectively. The goal of the PF is to simulate the posterior distribution $p(\boldsymbol{x}_{0:t}|\boldsymbol{y}_{1:t})$, which, according to Bayes' theorem, is given by

$$p(\mathbf{x}_{0:t}|\mathbf{y}_{1:t}) = \frac{p(\mathbf{y}_{1:t}|\mathbf{x}_{0:t})\, p(\mathbf{x}_{0:t})}{\int p(\mathbf{y}_{1:t}|\mathbf{x}_{0:t})\, p(\mathbf{x}_{0:t})\, d\mathbf{x}_{0:t}} \ . \quad (3)$$

Applying SIS, the state vector posterior distribution is approximated by

$$p(\boldsymbol{x}_{0:t}|\boldsymbol{y}_{1:t}) \approx \sum_{i=1}^{N_s} \tilde{\omega}_t^{(i)} \delta(\boldsymbol{x}_{0:t} - \boldsymbol{x}_{0:t}^{(i)}) \ , \quad (4)$$

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where the normalized importance weight $\tilde{\omega}_t^{(i)}$ is given by

$$\tilde{\omega}_t^{(i)} = \frac{\omega_t^{(i)}}{\sum_{j=1}^{N_s} \omega_t^{(j)}} \tag{5}$$

and where

$$\omega_t^{(i)} = \omega_{t-1}^{(i)} \frac{p(\boldsymbol{y}_t|\boldsymbol{x}_t^{(i)})p(\boldsymbol{x}_t^{(i)}|\boldsymbol{x}_{t-1}^{(i)})}{q(\boldsymbol{x}_t^{(i)}|\boldsymbol{x}_{0:t-1}^{(i)}, \boldsymbol{y}_{1:t})} \ . \tag{6}$$

Equation (6) is the importance weight update equation. The particles (or samples) $\boldsymbol{x}_t^{(i)}$ are drawn from the proposal distribution $q(\boldsymbol{x}_t|\boldsymbol{x}_{0:t-1}^{(i)}, \boldsymbol{y}_{0:t})$. In addition, $p(\boldsymbol{y}_t|\boldsymbol{x}_t^{(i)})$ and $p(\boldsymbol{x}_t^{(i)}|\boldsymbol{x}_{t-1}^{(i)})$ represent the system's likelihood and the state transition prior, respectively. The variable N_s denotes the number of particles. The proposal distribution is an arbitrary distribution which needs to have at least some support from the posterior distribution of the true state. In addition, it has been shown that the variance of the importance weights increases over time [6]. This implies that after a few iterations, all but one weight will converge to zero. When this occurs, a resampling scheme is introduced to solve this degeneracy problem. More specifically, in a resampling scheme, a new set of equally weighted particles are generated from the previous particles with large weights, while the particles with small weight are replaced. In other words, resampling discards the particles with small weights and focuses on the particles with more significant weights.

2.2. Particle Filter For Image Processing (Spatial Case)

To implement the PF, we employ a 2-D state space model similar to the ones used for Kalman filtering in [2, 12, 13].

2.2.1. The Image Model

We use the image model proposed in [13] since it is efficient in the sense of using only a small number of pixels:

$$\begin{aligned} \mathbb{I}(i,j) =& h_1\mathbb{I}(i,j-1) + h_2\mathbb{I}(i-1,j) \\ &+ h_3\mathbb{I}(i-1,j-1), \end{aligned} \tag{7}$$

where $\mathbb{I}(i,j)$ denotes the pixel at the i^{th} row and j^{th} column of the image. In (7), h_1, h_2, and h_3 are image parameters which can be estimated by various methods including, *e.g.*, least-squares. Next, the following state space model is constructed:

$$\mathbf{x}(i,j) = C\mathbf{x}(i,j-1) + Eu(i,j) + D\mathbf{w}(i,j), \tag{8}$$

$$y(i,j) = H\mathbf{x}(i,j) + v(i,j), \tag{9}$$

where

$$\mathbf{x} = \begin{bmatrix} \mathbb{I}(i,j) \\ \mathbb{I}(i,j-1) \\ \mathbb{I}(i-1,j+1) \\ \mathbb{I}(i-1,j) \end{bmatrix} C = \begin{bmatrix} h_1 & 0 & h_2 & h_3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T \quad D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}^T$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

The input term $u(i,j)$ is introduced as the recent estimate of pixel $\mathbb{I}(i-1,j+1)$. The variables $\mathbf{w}$ and v denote the process noise and the measurement noise, respectively.

2.2.2. The Spatial Particle Filter Algorithm

The first step in designing a particle filter is to choose a proposal distribution. While arbitrary in form, the proposal must be supported by the posterior distribution of the true system state and must be easy to sample. For PF-DWT we use a standard Kalman filter to provide the proposal distribution, since the image model is a linear system with non-Gaussian noise. Although the estimates delivered by the Kalman filter are not optimal in this case, they still provide the required support for the true posterior. The 2-D Kalman filter formulation is given by

$$\bar{\mathbf{x}}(i,j) = C\hat{\mathbf{x}}(i,j-1) + Eu(i,j) \tag{10}$$

$$\bar{P}(i,j) = CP(i,j-1)C^T + DQD^T \tag{11}$$

$$K(i,j) = \bar{P}(i,j)H^T[H\bar{P}(i,j)H^T + R]^{-1} \tag{12}$$

$$\hat{\mathbf{x}}(i,j) = \bar{\mathbf{x}}(i,j) + K(i,j)[y(i,j) - H\bar{\mathbf{x}}(i,j)] \tag{13}$$

$$P(i,j) = [I - K(i,j)H]\bar{P}(i,j) \ , \tag{14}$$

where R and Q denote the covariance of the process noise and measurement noise, respectively. Finally, the spatial particle filter algorithm is illustrated in Fig. 1 and is also summarized below.

1. Sequential Importance Sampling (SIS) Step:
   - At each iteration, calculate $\hat{\mathbf{x}}(i,j)$ and $P(i,j)$ according to (10)-(14).
   - Sample from the proposal distribution:

     $$x^{(i)}(i,j) \sim \mathcal{N}\left(\hat{\mathbf{x}}(i,j), P(i,j)\right) \ .$$

   - Evaluate the importance weights according to (6): calculate the transition prior, the likelihood and the proposal.
   - Normalize the importance weights using (5).
2. Resample and Update Step:
   - Generate a new set of particles $x^{i\star}(t)$ from $x^{(i)}(i,j)$, so that

     $$Pr\left(x^{i\star}(i,j) = x^{(j)}(i,j)\right) = \tilde{\omega}^{(j)}(i,j) \ .$$

   - The final estimate of $x(i,j)$ is given by

     $$x(i,j) \approx \frac{1}{N_s}\sum_{i=1}^{N_s} x^{i\star}(i,j) \ .$$

   - Update the proposal with the resampled particles.

2.3. Wavelet Thresholding for PF-DWT

We begin by applying the DWT to decompose the data into subbands. A threshold is then applied for noise removal. In the DWT domain, small coefficients in the high frequency subband are assumed to arise primarily from noise. The idea is to "zero out" the high frequency subband coefficients that are less than a particular threshold. These coefficients are used in an inverse wavelet transformation to reconstruct the data set. In this paper, we employ the universal threshold [3] [4] given as:

$$T = \sigma\sqrt{2\log_e N_d} \tag{15}$$

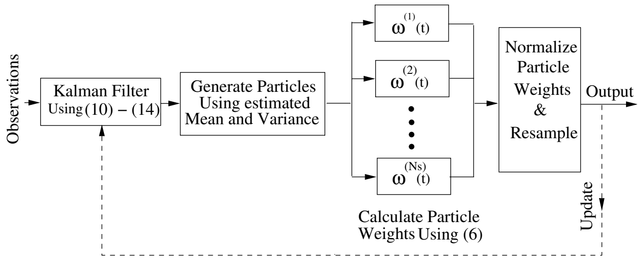


Fig. 1. Block Diagram of the Spatial Particle Filter

where N_d is the size of the data set. By definition, basic thresholding methods are hard thresholding:

$$\rho_T(x) = \begin{cases} x \ \ if|x| > T \\ 0 \ \ if|x| < T \end{cases} \tag{16}$$

and the soft thresholding:

$$\rho_T(x) = \begin{cases} x - T \ \ if \ x \geq T \\ x + T \ \ if \ x \leq -T \\ 0 \qquad \ \ if \ |x| \leq T \ . \end{cases} \tag{17}$$

Moreover, the universal thresholding method can be improved by using the translation invariant technique given in [3]. The basic idea of this method is to estimate the coefficients of all translations and take the average after a reverse translation. The final estimate is given by this average. More specifically, the coefficients $\tilde{F}^p$ of all the translated data, denoted by $X^p[n] = X[n-p]$, are calculated. By definition,

$$\tilde{F}^p = \sum_{m=0}^{N-1} \rho_T(X^p[m]) g_m \ , \tag{18}$$

where ρ_T is a hard or soft thresholding function. Then, these coefficients are shifted back and the averages are evaluated, which gives the final estimate as

$$\tilde{F}[n] = \frac{1}{N} \sum_{p=0}^{N-1} \tilde{F}^p[n+p] \ . \tag{19}$$

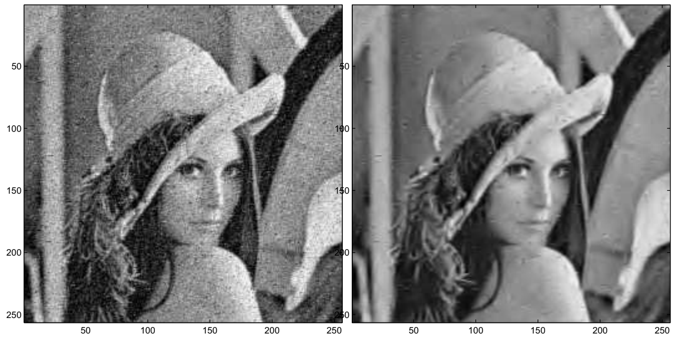
As indicated in [3], the translation invariant method produces a better denoised image than universal thresholding.

3. EXPERIMENTAL RESULTS

In this section, the standard 256×256 grayscale image "*Lena*" is used to assess the performance of the proposed hybrid denoising algorithm. The noisy image is generated by adding Gaussian noise with a variance of 200 and a Cauchy noise with pdf $f(x) = \theta/\pi(x^2 + \theta^2)$, where $\theta = 5$. The original and noise corrupted images are shown in Fig. 2(a) and (b), respectively. To construct the image model (7), the parameters $h_1 = 0.815493$, $h_2 = 0.489516$, $h_3 = -0.308866$ were employed. These values were estimated by using the least squares method [13].

The two-stage PF-DWT hybrid denoising process is carefully tailored to operate synergistically. In the first stage, a PF which uses Kalman filter state estimates as its proposal is utilized to suppress the heavy tailed noise. After the this stage, the remaining noise is "Gaussian-like" in character and is attenuated by wavelet thresholding. In the second stage, translation invariant soft/hard thresholding is applied to the PF result to achieve further noise removal. Two examples of images restored in this way are shown in Fig. 2 and 3. As indicated in Fig. 2(c), the PF removes most of the heavy-tailed noise. The remaining noise, which is observed to have a "Gaussian-like" distribution, can be removed using the wavelet thresholding method. The denoised image obtained via PF and translation invariant hard thresholding is shown in Fig. 2(d). As a second example, the grayscale image "*cameraman*" is also used to test the performance of the proposed method. The original, noisy, and denoised images are show in Fig. 3(a)-(d), respectively. These results show that the restored images Fig. 2(d) and Fig. 3(c), (d) are of remarkable visual quality. In addition, as a quantitative performance measure, the improved signal to noise ratio (ISNR) of the proposed hybrid method are also shown in Fig. 2 and Fig. 3.

(a)The Original Image (b) The Noisy Image

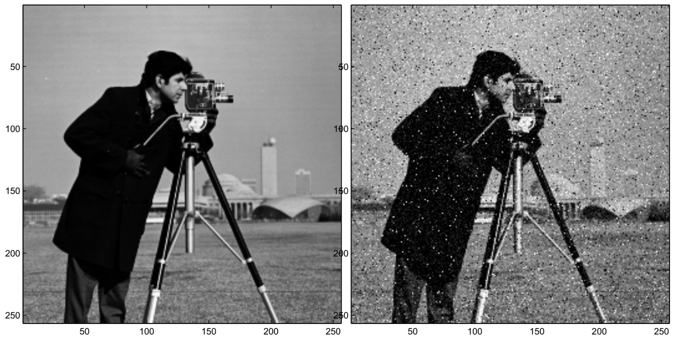


(c) Restored Image Using PF (ISNR= 3.1991 dB) (e) PF with Translation Invariant Hard Thresholding (ISNR= 6.7126 dB)

Fig. 2. This figure shows: (a) the original image, (b) the noisy image, (c) the result obtained by only applying the particle filter, (d) the restored images obtained by the PF with the translation invariant thresholding.

4. CONCLUSIONS AND FUTURE WORK

Images transmitted over modern packet switched wireless channels often contain both Gaussian and heavy-tailed noise. In this paper, a PF-DWT hybrid denoising method was proposed for restor-



(a)The Original Image (b) The Noisy Image

(c) PF with Translation Invariant Hard Thresholding ISNR= 5.4722 dB

(d) PF with Translation Invariant Soft Thresholding ISNR= 3.4959 dB

Fig. 3. As another example, this figure shows: (a) the original image, (b) the noisy image, (c) and (d) the restored images obtained by PF with translation invariant hard/soft thresholding.

ing an image contaminated by an inhomogeneous mixture of both types of noise. The simulation results presented here demonstrate that the proposed method can effectively recover the original images and merits future study. Various techniques including the *BayesShrink* algorithm [14] have been proposed recently for calculating improved thresholds. Based on the observation that the wavelet coefficients in a subband can be approximated by a generalized Gaussian distribution (GGD), the improved threshold is given by $\hat{T} = \hat{\sigma}^2/\hat{\sigma}_X$, where $\hat{\sigma}^2$ and $\hat{\sigma}_X$ are the estimated noise variance and standard deviation of the GGD wavelet coefficients, respectively. Important future work will include incorporating this technique into the PF-DWT approach in the near future.

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