# RECENT ADVANCES IN MULTI-COMPONENT AM-FM IMAGE MODELING

*David S. Harding, Joseph P. Havlicek, and Alan C. Bovik*

Laboratory for Vision Systems, University of Texas, Austin, TX 78712-1084

## ABSTRACT

We compute multi-component AM-FM representations for images using a new statistical component model. Consistent with processing known to occur in biological vision systems, the individual components of a multi-partite image are isolated on a spatio-spectrally localized basis using a multiband filterbank comprising frequency and orientation selective channels. Estimates of the modulating functions of each component are derived from the channel responses using localized nonlinear operators followed by optimal MMSE estimators. We also demonstrate image reconstruction from the representation.

## 1. INTRODUCTION

The efficacy of AM-FM modeling techniques for analyzing and characterizing nonstationary images has been well established [1–3]. These techniques are most useful when $t(\mathbf{x})$, the images of interest, may be accurately represented as the real part of a sum of locally coherent components of the form $s(\mathbf{x}) = a(\mathbf{x})\exp[j\varphi(\mathbf{x})]$, where $\mathbf{x} = (x_1, x_2)$, and $a,\ \varphi : \mathbb{R}^2 \to \mathbb{R}$. Individually, these components may be demodulated using a localized nonlinear operator [1]. However, int the presence of multiple components, this approach fails due to cross-component interferece. To circumvent this, a bank of wavelet-like Gabor filters is used to separate the components prior to demodulation. The design of such a filterbank is described elsewhere [3]. Demodulation is then accomplished using the approximate filtered algorithm

$$\nabla\varphi(\mathbf{x}) \approx \nabla\widehat{\varphi}(\mathbf{x}) = \mathrm{Re}\left[\frac{\nabla t_m(\mathbf{x})}{j t_m(\mathbf{x})}\right], \qquad (1)$$

$$a(\mathbf{x}) \approx \widehat{a}(\mathbf{x}) = \left|\frac{t_m(\mathbf{x})}{G_m\left[\nabla\widehat{\varphi}(\mathbf{x})\right]}\right|, \qquad (2)$$

where $t_m(\mathbf{x}) = t(\mathbf{x}) * g_m(\mathbf{x})$ is the response of the $m$th channel, $g_m(\mathbf{x})$ is the $m$th channel filter, with Fourier transform $G_m[\cdot]$. The approximation errors in the numerator and denominator of (1) are tightly bounded by a *quasi-eigenfunction theorem* [1,2] involving certain functional norms of $g_m(\mathbf{x})$, $a(\mathbf{x})$, and $\nabla\varphi(\mathbf{x})$. By applying (1), (2) to the responses of *all* filterbank channels, estimates of the modulating functions of *all* components are produced at *all* pixels.

## 2. STATISTICAL STATE-SPACE COMPONENT MODEL

Given an initial phase sample, components are determined by their amplitude $a(\mathbf{x})$ and horizontal and vertical frequencies $\varphi^x(\mathbf{x}) = \frac{\partial}{\partial x}\varphi(\mathbf{x})$ and $\varphi^y(\mathbf{x}) = \frac{\partial}{\partial y}\varphi(\mathbf{x})$. We introduce an artificial temporal causality relationship between points in the sampled image domain by mapping them to a discrete 1D lattice according to a path function $\mathcal{O} : \mathbf{x} \longmapsto k$. This reparamaterization maps the modulating functions of a component according to $a(\mathbf{x}) \overset{\mathcal{O}}{\longmapsto} a(k)$, $\varphi^x(\mathbf{x}) \overset{\mathcal{O}}{\longmapsto} \varphi^x(k)$, and $\varphi^y(\mathbf{x}) \overset{\mathcal{O}}{\longmapsto} \varphi^y(k)$. Each of the modulating functions can then be expanded in first-order Taylor series about a lattice point $k$. The series for $a(k)$ is

$$a(k) = a(k-1) + a'(k-1) + \int_{k-1}^{k}(k-\rho)\frac{\partial^2}{\partial\rho^2}a(\rho)d\rho \qquad (3)$$

and the series for $\varphi^x(\mathbf{x})$, $\varphi^y(\mathbf{x})$ are similar.

Modeling the integral in (3) as a noise process $u_a$ (similarly for $u_{\varphi_x}$ and $u_{\varphi_y}$), we arrive at the component state-space model

$$X(k+1) = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A \end{bmatrix} X(k) + Iu(k) \qquad (4)$$

$$Y(k) = C(k)X(k), \qquad (5)$$

where the state vector $X(k) = [a(k) \quad a'(k) \quad \varphi^x(k) \quad \varphi^{x\prime}(k) \quad \varphi^y(k) \quad \varphi^{y\prime}(k)]^T$,

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \qquad (6)$$

This research was supported in part by a grant from the Texas Advanced Research Projects Agency and by the Air Force Office of Scientific Research, Air Force Systems Command, USAF, under grant number F49620-93-1-0307.

$u(k) = \begin{bmatrix} 0 & u_a(k) & 0 & u_{\varphi_x}(k) & 0 & u_{\varphi_y}(k) \end{bmatrix}^T$, $Y(k) = \begin{bmatrix} a(k) & \varphi^x(k) & \varphi^y(k) \end{bmatrix}^T$, and $C(k)$ is the output gain matrix. We relate the quantities in the state-space model output vector $Y(k)$, which are the exact modulating functions, to the observations obtained from the filtered demodulation algorithm (1), (2) by modeling the estimation errors with additive uncorrelated noise processes $n_a(k)$, $n_{\varphi_x}(k)$, and $n_{\varphi_y}(k)$.

Hence we describe the estimates of each component's modulating functions as noisy observations of an affine function of the state vector of a linear system driven by white noise, and can therefore design Kalman filters to obtain optimal estimates of the modulating functions of each component as the image is traversed along the path $\mathcal{O}$. These filters produce unbiased estimates $\widetilde{a}(k)$, $\widetilde{\varphi^x}(k)$, $\widetilde{\varphi^y}(k)$, which are optimal in the MMSE sense over the class of linear estimators. A block diagram of the complete multi-component demodulation algorithm including the Kalman-based track processor is shown in Figure 1.

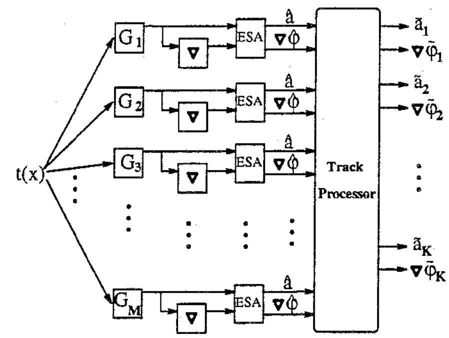


Figure 1: System for computing the multi-component AM-FM representation.

## 3. EXAMPLE

In Figure 2, we compute the multi-component AM-FM representation of a synthetically generated two-component image exhibiting significant 2-D nonstationarity, and then reconstruct. The original image is shown in the (a) part of the figure. To avoid edge effects from the channel filters, tracking and reconstruction were not performed on the outside 16 rows and columns of the image. The presence of two components was correctly identified by the track processor. The reconstructed components were summed to obtain the reconstructed image shown in Figure 2 (b). Note that the reconstruction is visually indistinguishable from the original.

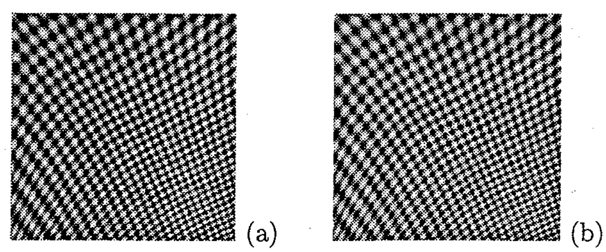


Figure 2: Multi-component AM-FM representation and reconstruction of a synthetic image. (a) Nonstationary two-component synthetic image. (b) Reconstructed image.

## 4. FUTURE WORK

Important future work remaining in this area includes overcoming the extremely difficult problems in treating complicated natural images. In particular, this will involve developing an approach for tracking many components closely spaced in frequency over irregularly shaped regions of support, and fully characterizing the classes of images which can be represented.

## 5. REFERENCES

[1] J. P. Havlicek and A. C. Bovik, "Multi-component AM-FM image models and wavelet-based demodulation with component tracking", in *Proc. IEEE Int'l. Conf. Image Proc.*, Austin, TX, November 13-16 1994, pp. I41 – I45.

[2] J. P. Havlicek, A. C. Bovik, and P. Maragos, "Modulation models for image processing and wavelet-based image demodulation", in *Proc. 26th IEEE Asilomar Conf. Signals, Syst., Comput.*, Pacific Grove, CA, October 26 - 28 1992, pp. 805–810.

[3] J. P. Havlicek and A. C. Bovik, "AM-FM models, the analytic image, and nonlinear demodulation techniques", Tech. Rept. TR-95-001, Center for Vision and Image Sciences, The University of Texas at Austin, March 1995.