# Designing Perceptually-Based Image Filters in the Modulation Domain

Joseph P. Havlicek

*School of Electrical and Computer Engineering*
*University of Oklahoma*

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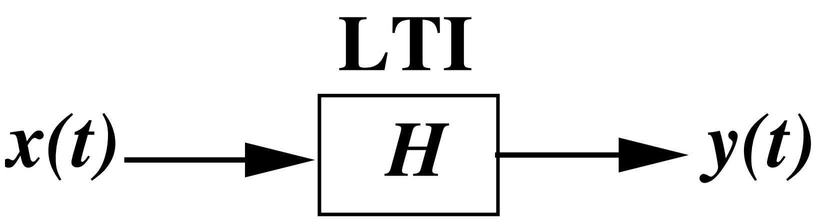
# Eigenfunctions of LTI System

- For any $\omega_0 \in \mathbb{R}$, the signal $x(t) = e^{j\omega_0 t}$ is an Eigenfunction of any 1-D continuous-time LTI system.




$$x(t) = e^{j\omega_0 t}$$

$$\begin{aligned}
y(t) &= h(t) * x(t) = h(t) * e^{j\omega_0 t} \\
&= \int_{\mathbb{R}} h(\tau) e^{j\omega_0 (t-\tau)} d\tau \\
&= e^{j\omega_0 t} \underbrace{\int_{\mathbb{R}} h(\tau) e^{j\omega_0 \tau} d\tau}_{\text{a number}} \\
&= H(\omega_0) e^{j\omega_0 t} \\
&= |H(\omega_0)| e^{\{j\omega_0 t + \arg H(\omega_0)\}}
\end{aligned}$$

# Representing Signals as Sums of Eigenfunctions

- We use the Fourier transform to write an arbitrary input $x(t)$ as a sum of Eigenfunctions:

$$x(t) = \frac{1}{2\pi} \int_{\mathbb{R}} X(\omega) e^{j\omega t} dt.$$

- The action of the LTI system $H$ is that each term in the sum gets multiplied by a corresponding Eigenvalue $H(\omega)$:

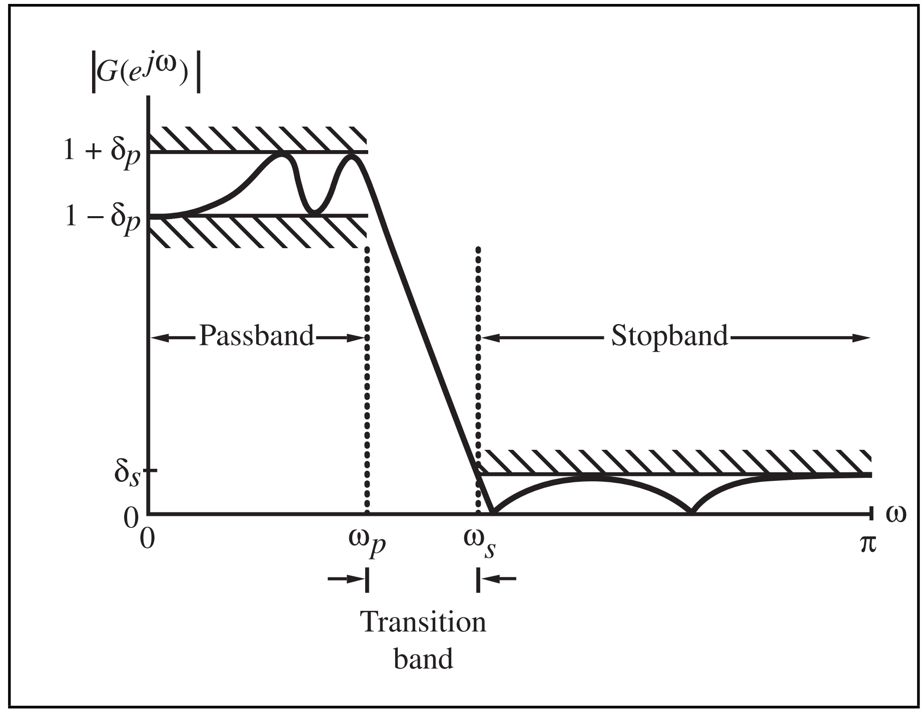
$$y(t) = \frac{1}{2\pi} \int_{\mathbb{R}} H(\omega) X(\omega) e^{j\omega t} dt.$$

  - Each term is scaled by $|H(\omega)|$ and shifted by $\arg H(\omega)$.

# The LTI Filter Design Problem

- Design the Eigenvalues $H(\omega)$, e.g., the *frequency response*, to achieve some desired signal processing goal.

- What kinds of problems is this approach good for?
  - attenuate additive noise with a stationary spectrum.
  - in music: boost the bass and attenuate the midrange – related to how human hearing *perceives* the signal.

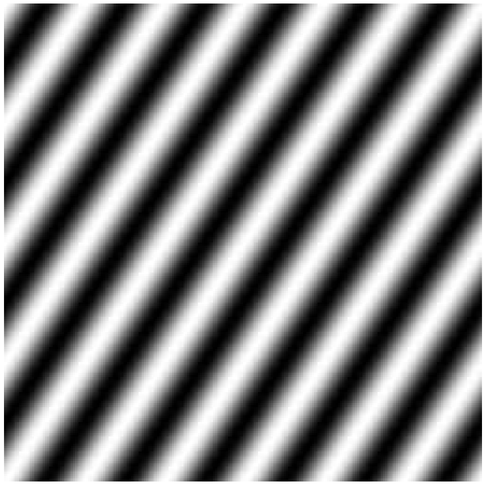


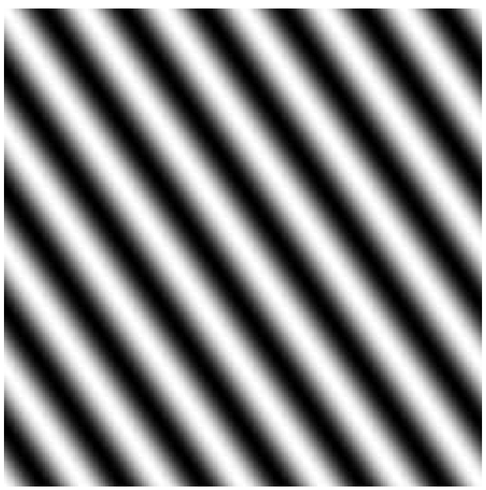
# Human Auditory Perception

- But is human auditory perception *really* very closely related to the Eigenfunction representation?

# What About Human Vision?

- Are these Eigenfunctions

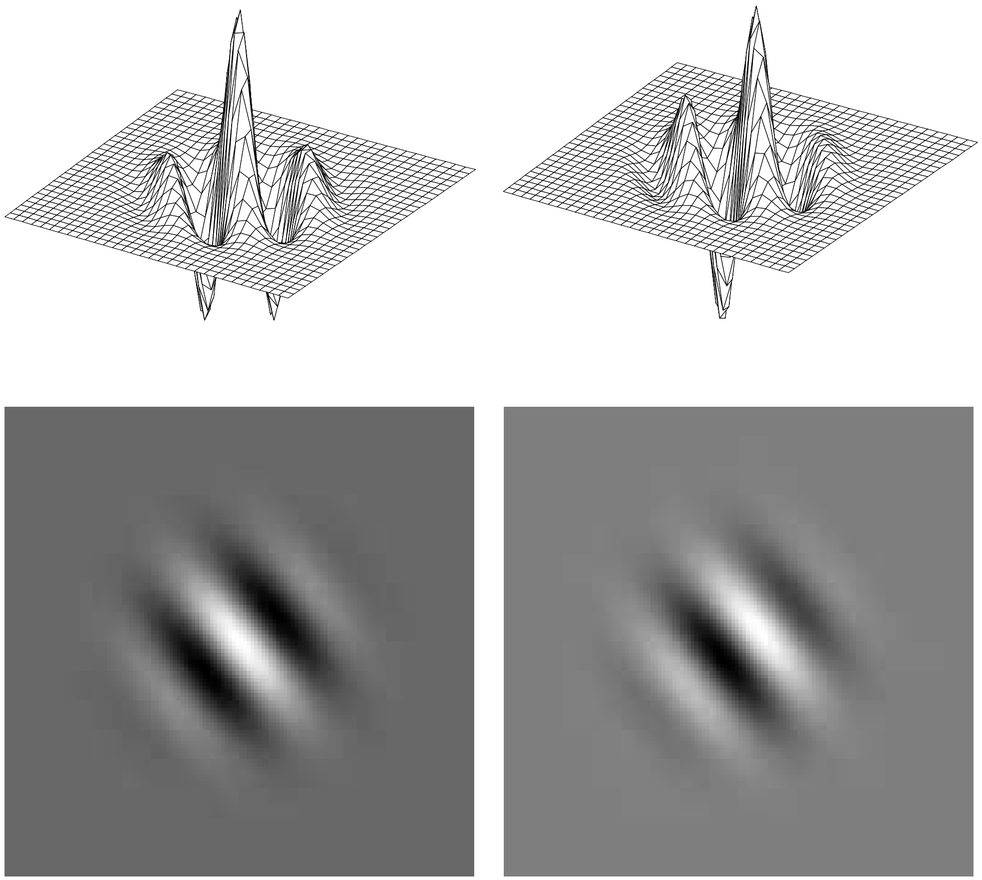




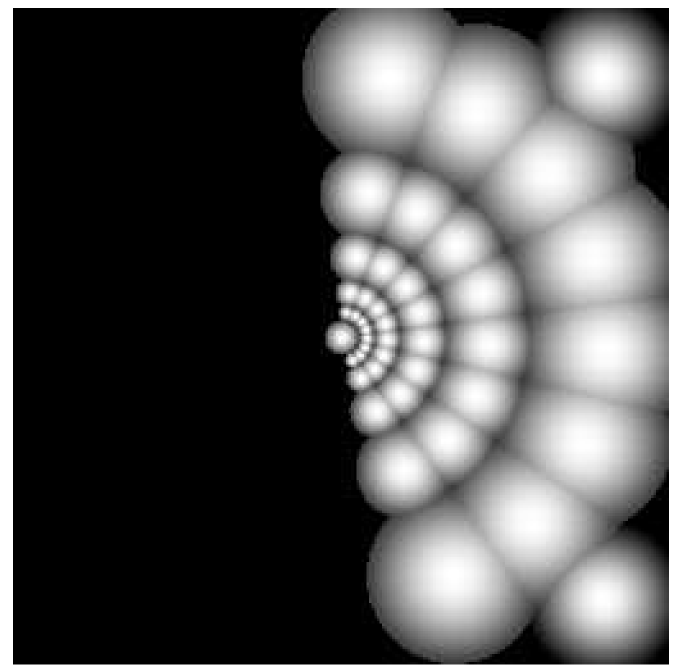
- Closely related to human visual perception of this?

# Gabor Aspects of Mammalian Biological Vision

- Complex Gabor filter:



- Biologically motivated Gabor filter bank:



# Modulation Domain Signal Representation

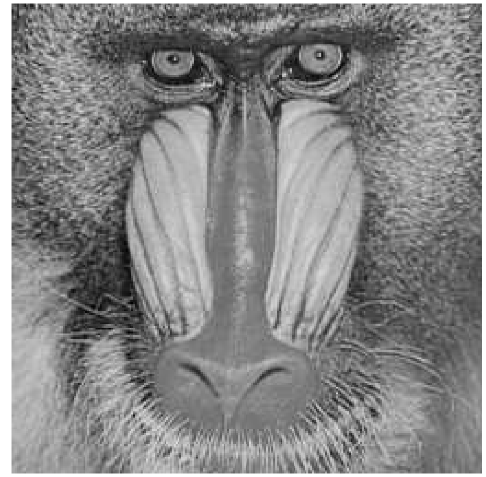
- A nonstationary image component: $t_k(\mathbf{x}) = a_k(\mathbf{x}) \exp[j\varphi_k(\mathbf{x})]$.
- Modulation domain signal model:

$$t(\mathbf{x}) = \sum_{k=1}^{K} t_k(\mathbf{x}) = \sum_{k=1}^{K} a_k(\mathbf{x}) \exp[j\varphi_k(\mathbf{x})].$$

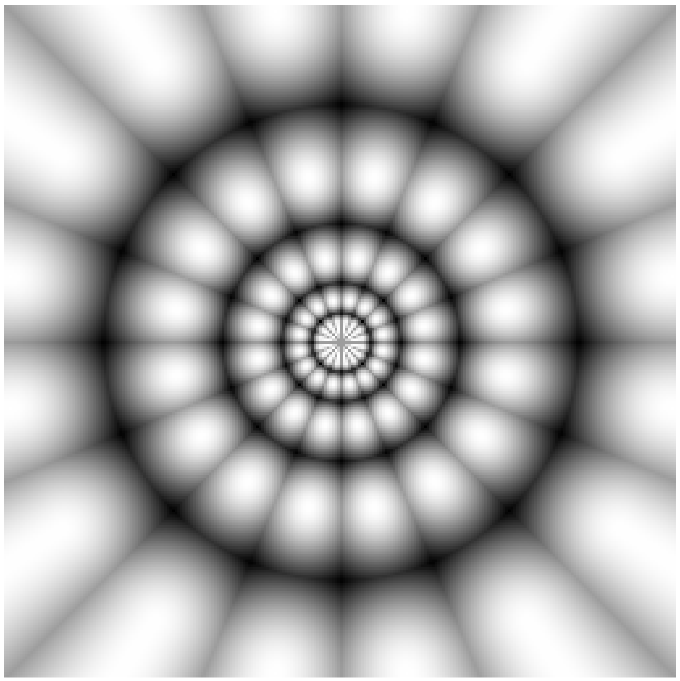
- AM: $a_k(\mathbf{x})$
  - local texture contrast.
- FM: $\nabla\varphi_k$
  - local texture orientation and granuliarity.

# AM-FM Signal Components

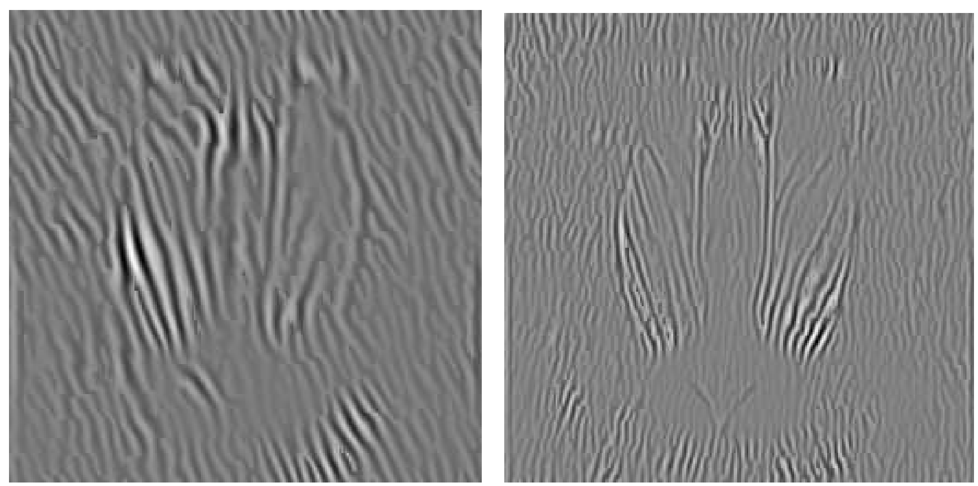
- Image:



- Steerable Pyramid:



- AM-FM Components:



# Nonlinear Demodulation Algorithm

- AM-FM image component:

$$t_k(\mathbf{x}) = a_k(\mathbf{x}) \exp[j\varphi_k(\mathbf{x})]$$

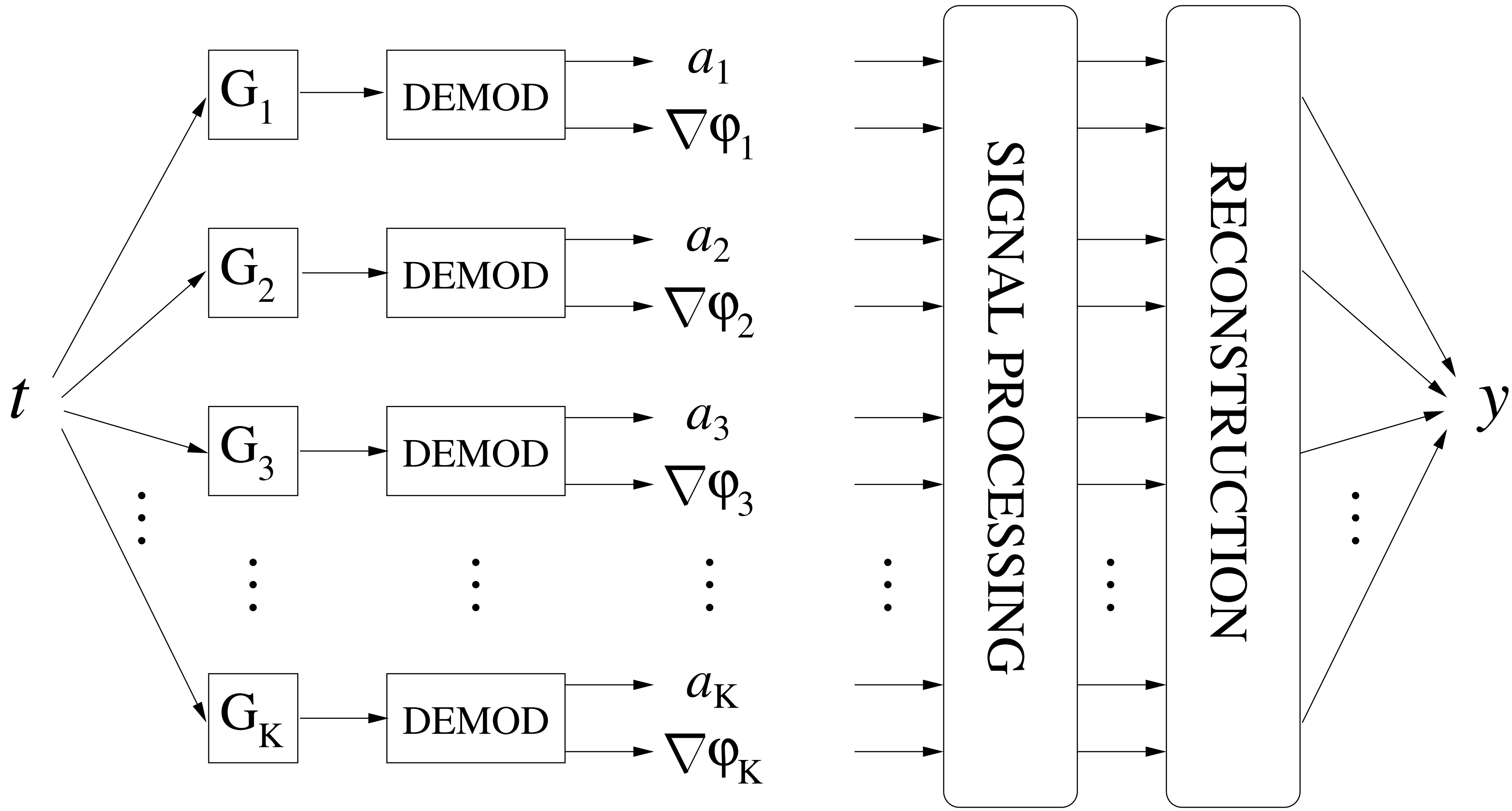
- Interpolate $t_k(\mathbf{x})$ with a cubic tensor product spline.
- FM: local texture orientation and granuliarity:

$$\nabla\varphi_k(\mathbf{x}) = \mathrm{Re}\left[\frac{\nabla t_k(\mathbf{x})}{j t_k(\mathbf{x})}\right]$$

- AM: local texture contrast:

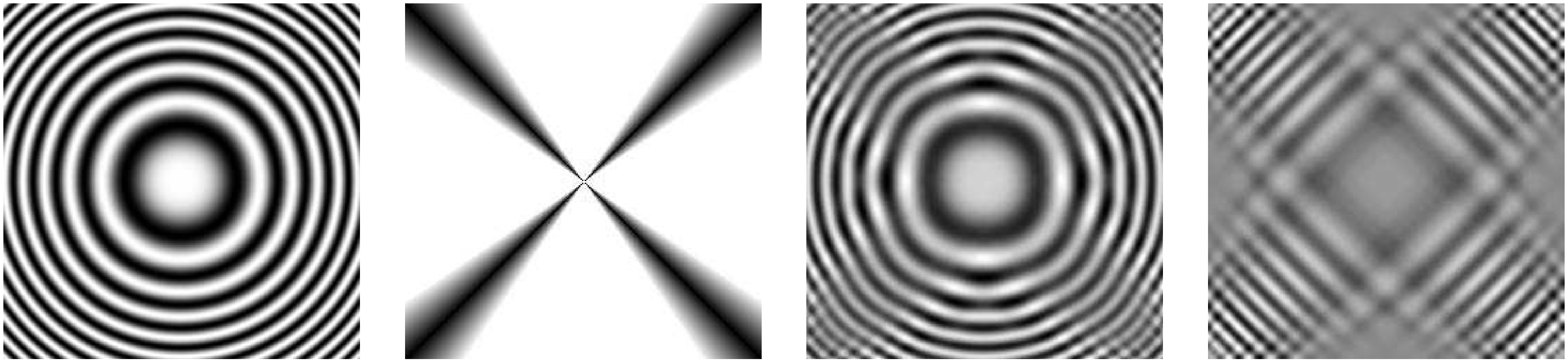
$$a_k(\mathbf{x}) = |t_k(\mathbf{x})|$$

# Modulation Domain Signal Processing

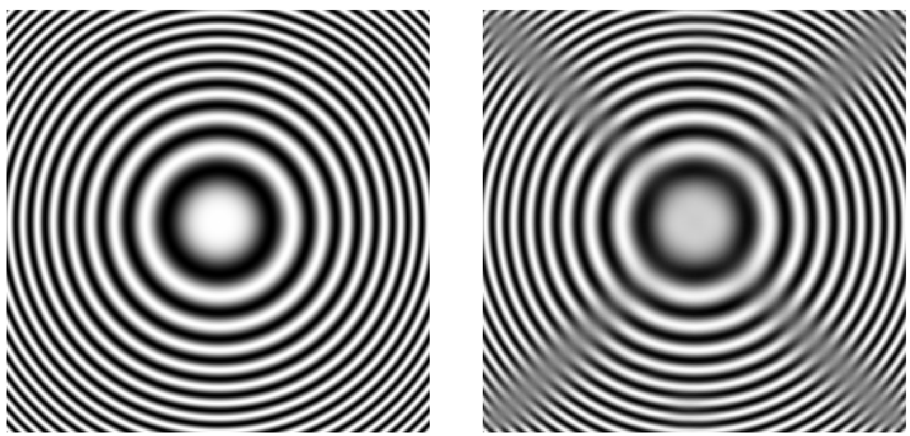


# Orientation Selective Attenuation

- Best LTI filtering result:

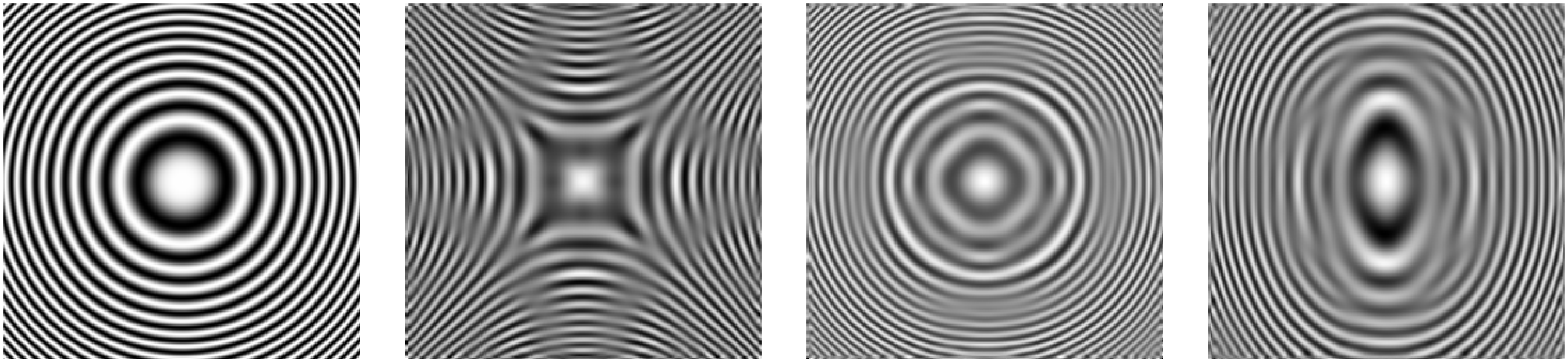


- Modulation domain filtering result:

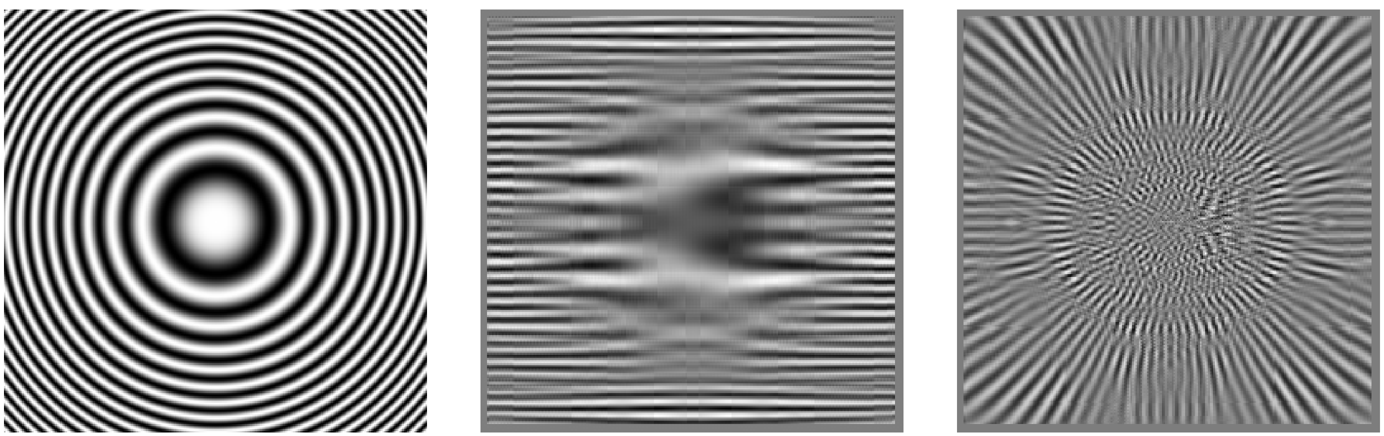


# FM Processing Examples

- Least squares phase reconstruction:



- Spline-based phase reconstruction:



# Lena Example

# Barbara Example