

# Designing Perceptually-Based Image Filters in the Modulation Domain

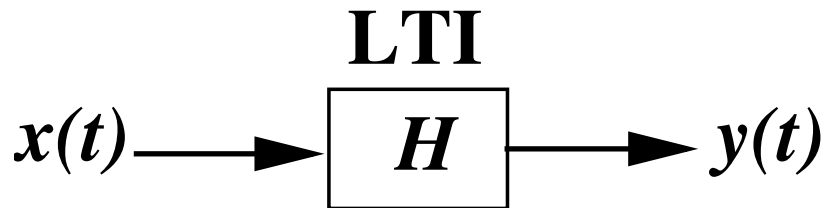
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# Eigenfunctions of LTI System

- ▶ For any  $\omega_0 \in \mathbb{R}$ , the signal  $x(t) = e^{j\omega_0 t}$  is an Eigenfunction of any 1-D continuous-time LTI system.



$$x(t) = e^{j\omega_0 t}$$

$$\begin{aligned} y(t) &= h(t) * x(t) = h(t) * e^{j\omega_0 t} \\ &= \int_{\mathbb{R}} h(\tau) e^{j\omega_0(t-\tau)} d\tau \\ &= e^{j\omega_0 t} \underbrace{\int_{\mathbb{R}} h(\tau) e^{j\omega_0 \tau} d\tau}_{\text{a number}} \\ &= H(\omega_0) e^{j\omega_0 t} \\ &= |H(\omega_0)| e^{\{j\omega_0 t + \arg H(\omega_0)\}} \end{aligned}$$

# Representing Signals as Sums of Eigenfunctions

- ▶ We use the Fourier transform to write an arbitrary input  $x(t)$  as a sum of Eigenfunctions:

$$x(t) = \frac{1}{2\pi} \int_{\mathbb{R}} X(\omega) e^{j\omega t} dt.$$

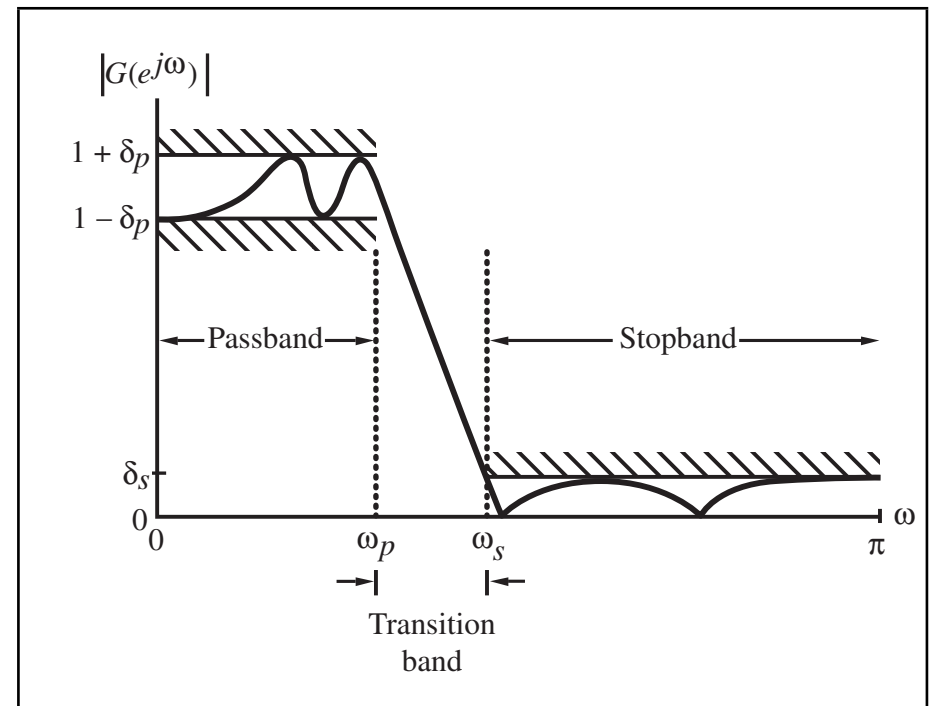
- ▶ The action of the LTI system  $H$  is that each term in the sum gets multiplied by a corresponding Eigenvalue  $H(\omega)$ :

$$y(t) = \frac{1}{2\pi} \int_{\mathbb{R}} H(\omega) X(\omega) e^{j\omega t} dt.$$

- Each term is scaled by  $|H(\omega)|$  and shifted by  $\arg H(\omega)$ .

# The LTI Filter Design Problem

- ▶ Design the Eigenvalues  $H(\omega)$ , e.g., the *frequency response*, to achieve some desired signal processing goal.
- ▶ What kinds of problems is this approach good for?
  - attenuate additive noise with a stationary spectrum.
  - in music: boost the bass and attenuate the midrange – related to how human hearing *perceives* the signal.



# Human Auditory Perception

- ▶ But is human auditory perception *really* very closely related to the Eigenfunction representation?

Allegro assai

The image shows a musical score for an orchestra. The tempo is marked 'Allegro assai'. The time signature is 2/4. The key signature has one flat (B-flat). The instruments listed are Tromba, Flauto, Oboe, Violino, Violino 1 di ripieno, Violino 2 di ripieno, Viola di ripieno, Violone di ripieno, and Violoncello e Cembalo. The Tromba part has trills (tr) in the first three measures. The Violoncello e Cembalo part has a rhythmic pattern of eighth notes. The other instruments are mostly silent, with some activity in the Oboe and Violoncello e Cembalo parts in the final measure.

Tromba

Flauto

Oboe

Violino

Violino 1 di ripieno

Violino 2 di ripieno

Viola di ripieno

Violone di ripieno

Violoncello e Cembalo

# What About Human Vision?

- ▶ Are these Eigenfunctions

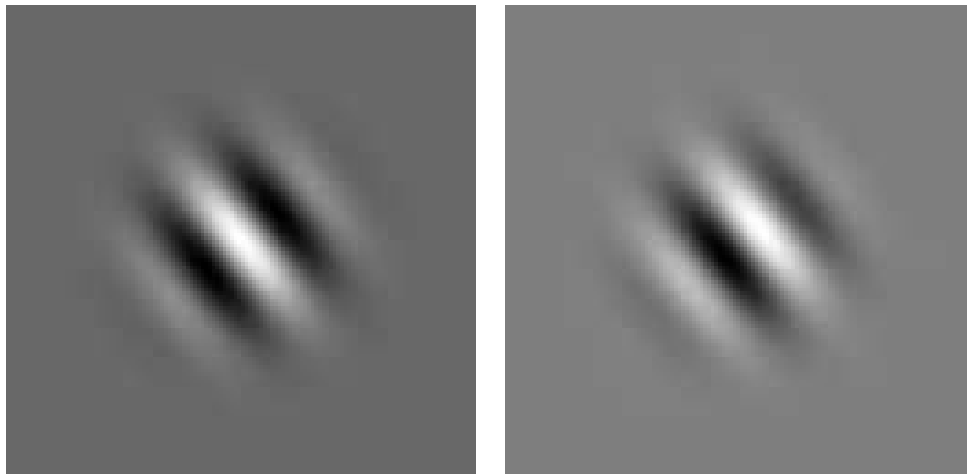
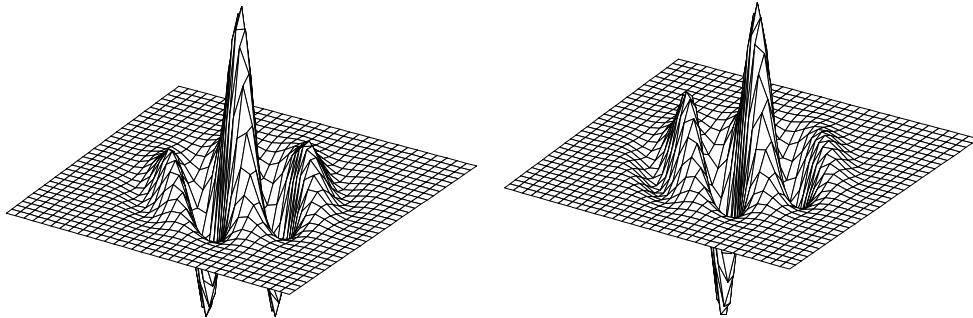


- ▶ Closely related to human visual perception of this?

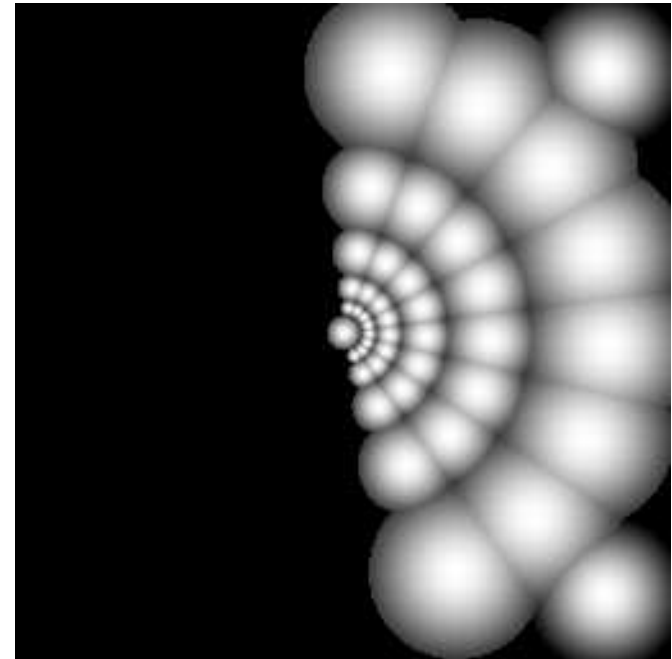


# Gabor Aspects of Mammalian Biological Vision

- ▶ Complex Gabor filter:



- ▶ Biologically motivated Gabor filter bank:



# Modulation Domain Signal Representation

- ▶ A nonstationary image component:  $t_k(\mathbf{x}) = a_k(\mathbf{x}) \exp[j\varphi_k(\mathbf{x})]$ .
- ▶ Modulation domain signal model:

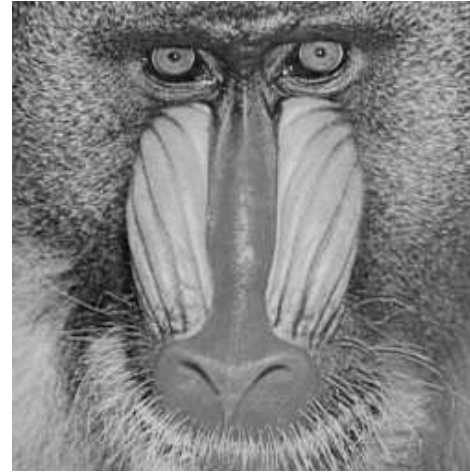
$$t(\mathbf{x}) = \sum_{k=1}^K t_k(\mathbf{x}) = \sum_{k=1}^K a_k(\mathbf{x}) \exp[j\varphi_k(\mathbf{x})].$$

- ▶ AM:  $a_k(\mathbf{x})$ 
  - local texture contrast.
- ▶ FM:  $\nabla\varphi_k$ 
  - local texture orientation and granularity.

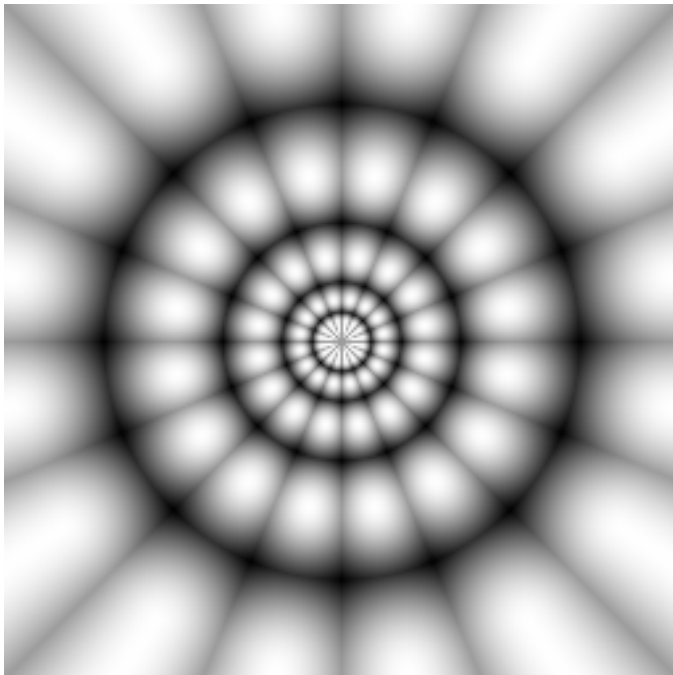


# AM-FM Signal Components

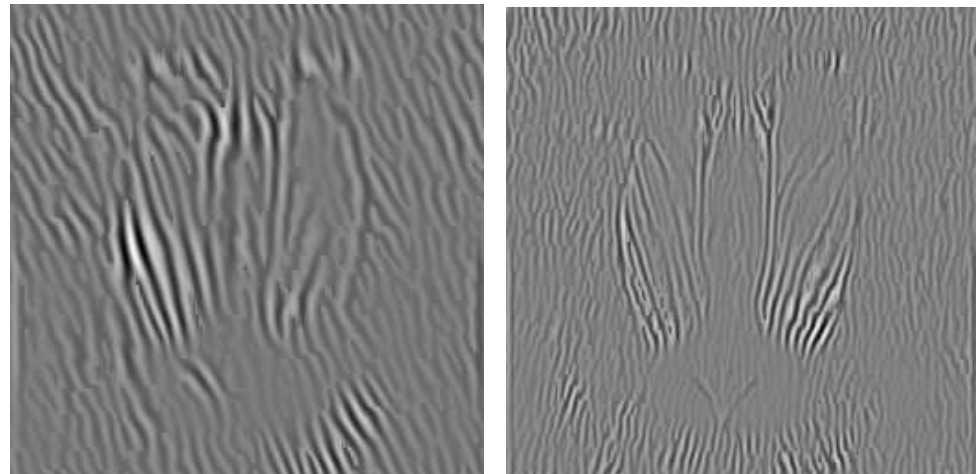
▶ Image:



▶ Steerable Pyramid:



▶ AM-FM Components:



# Nonlinear Demodulation Algorithm

- ▶ AM-FM image component:

$$t_k(\mathbf{x}) = a_k(\mathbf{x}) \exp[j\varphi_k(\mathbf{x})]$$

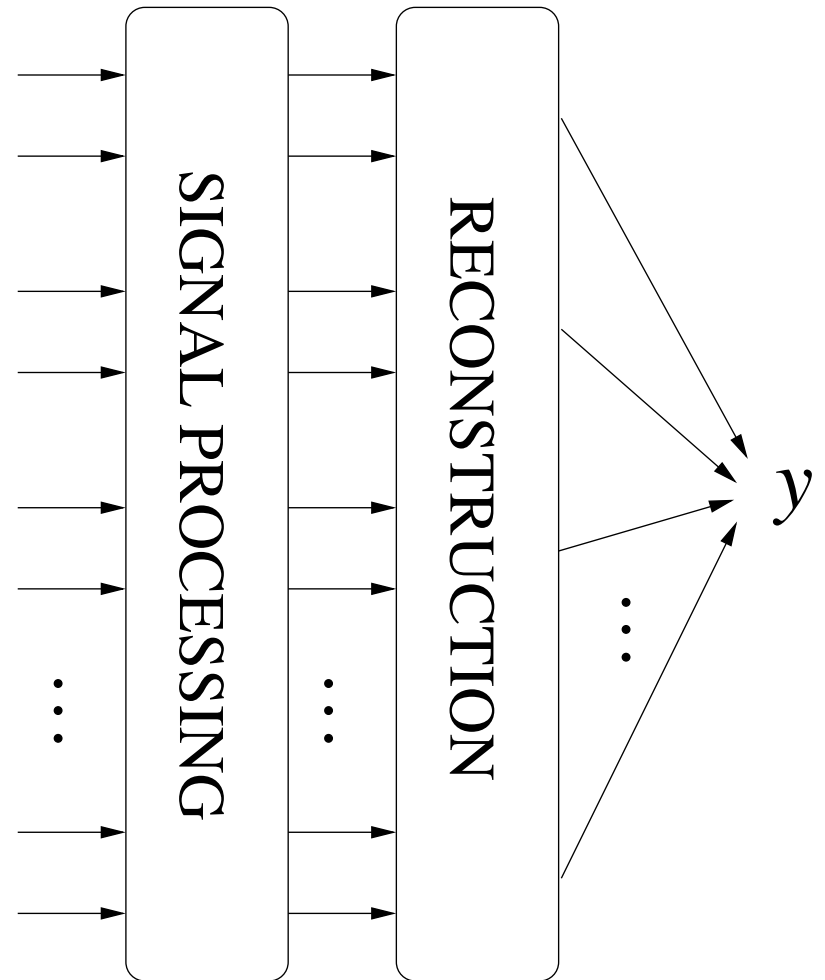
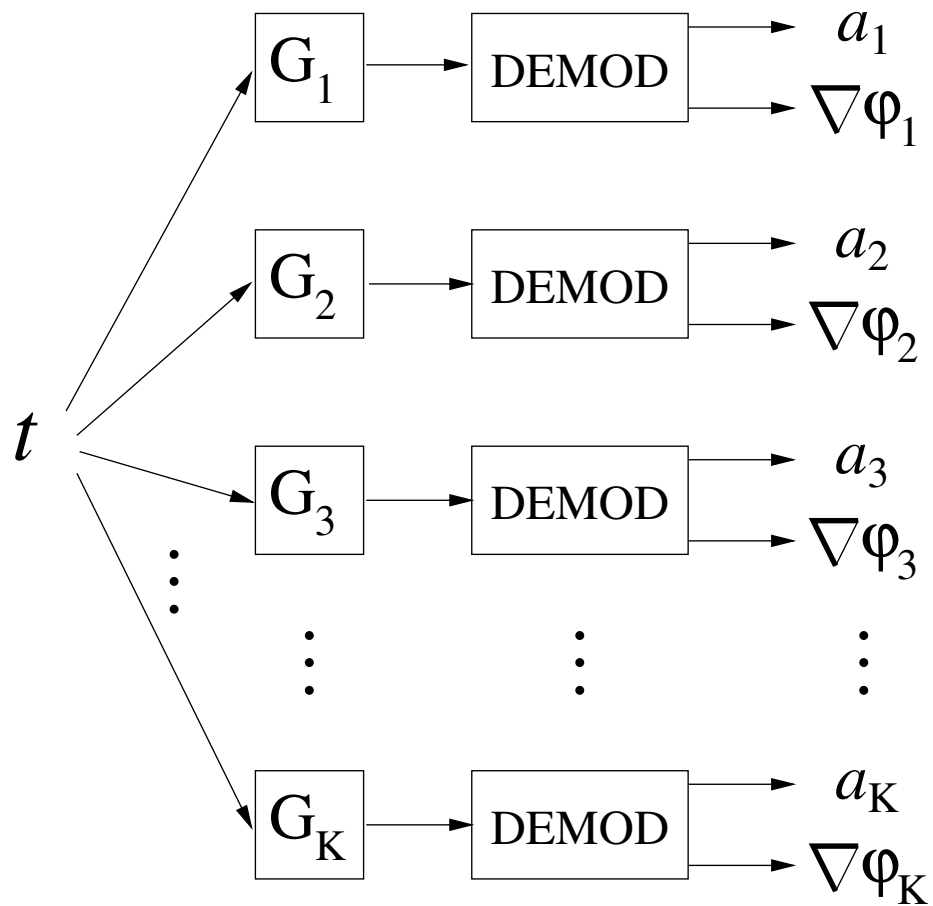
- ▶ Interpolate  $t_k(\mathbf{x})$  with a cubic tensor product spline.
- ▶ FM: local texture orientation and granularity:

$$\nabla\varphi_k(\mathbf{x}) = \operatorname{Re} \left[ \frac{\nabla t_k(\mathbf{x})}{jt_k(\mathbf{x})} \right]$$

- ▶ AM: local texture contrast:

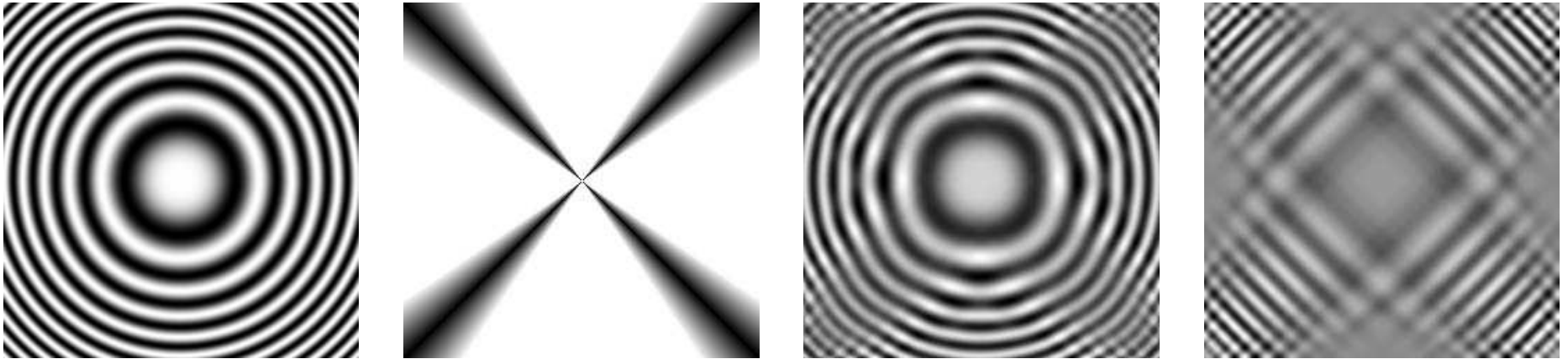
$$a_k(\mathbf{x}) = |t_k(\mathbf{x})|$$

# Modulation Domain Signal Processing

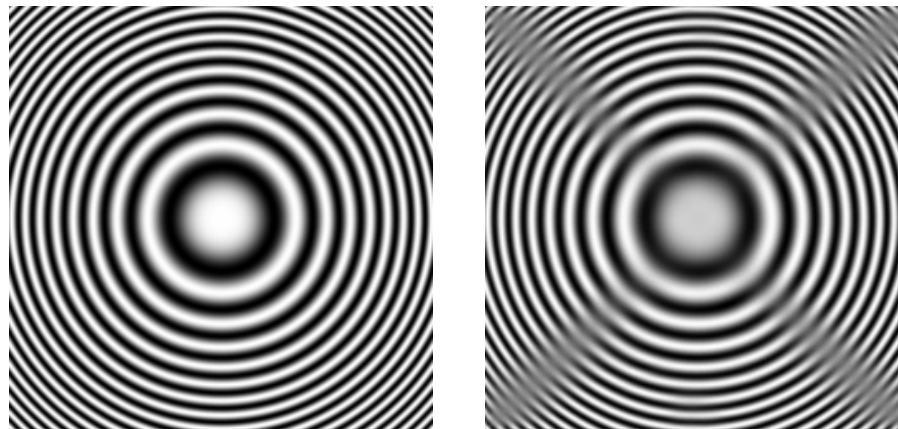


# Orientation Selective Attenuation

- Best LTI filtering result:

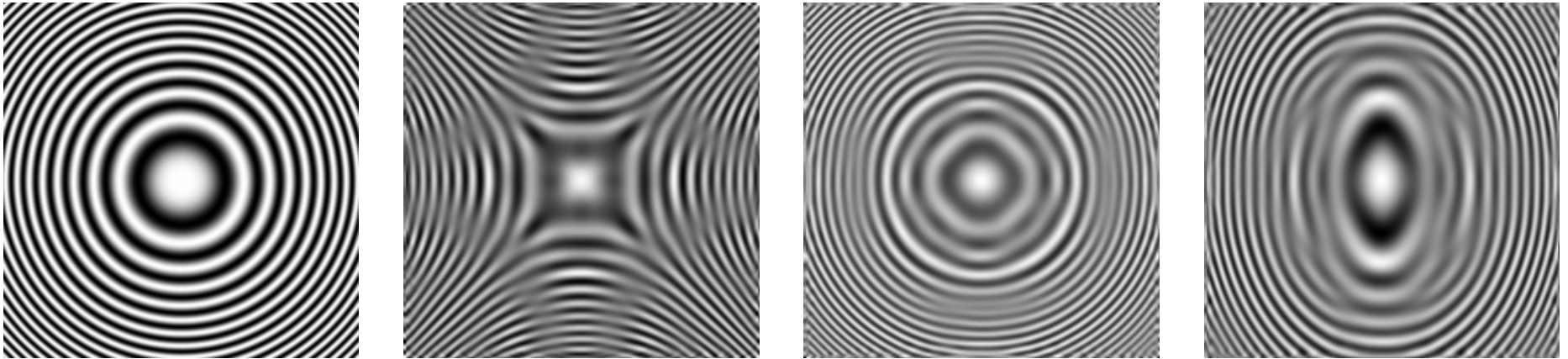


- Modulation domain filtering result:

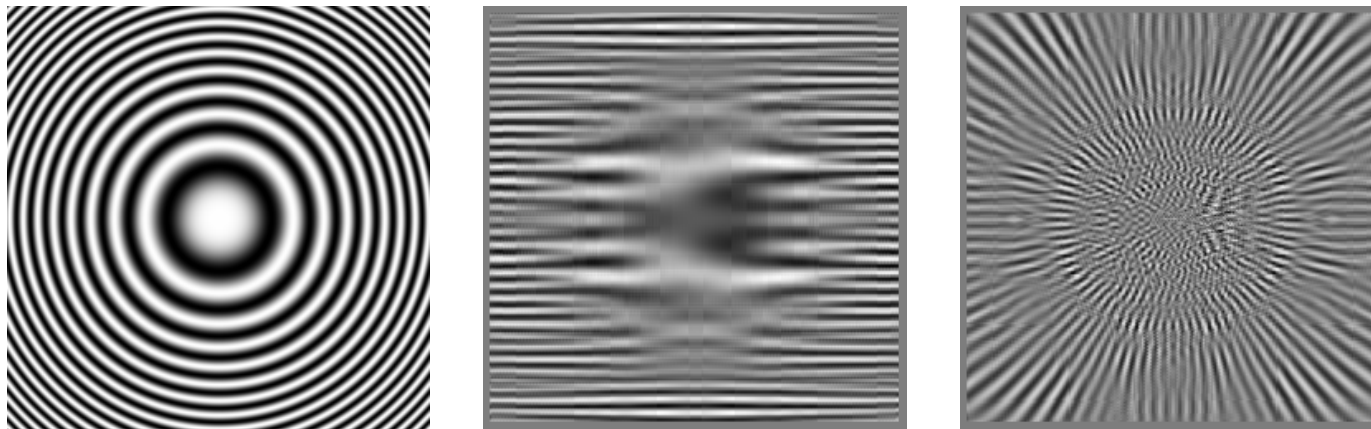


# FM Processing Examples

- Least squares phase reconstruction:



- Spline-based phase reconstruction:



# Lena Example



# Barbara Example

