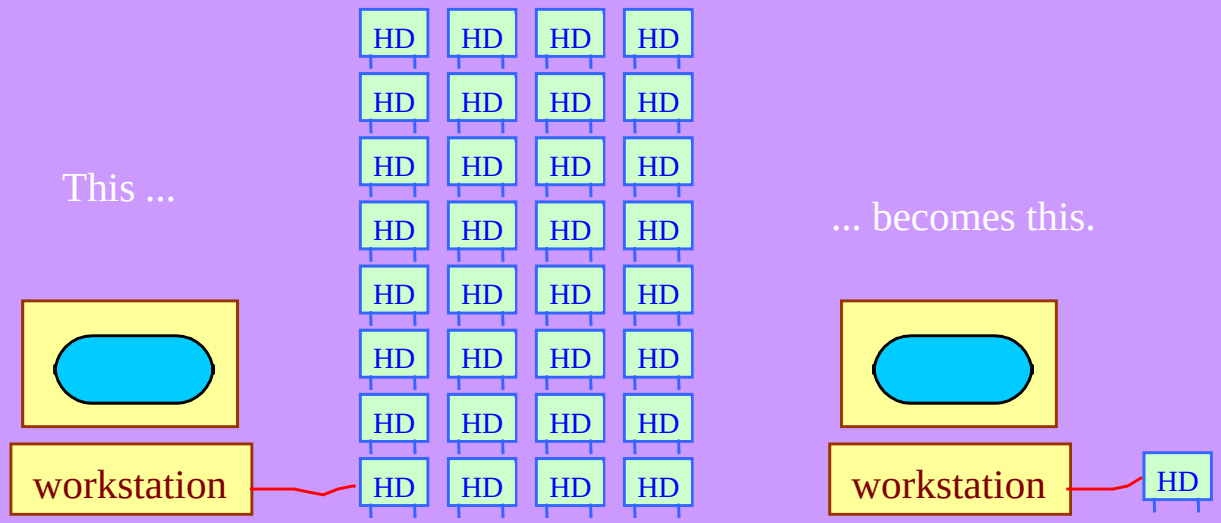
Module 7
Image Compression

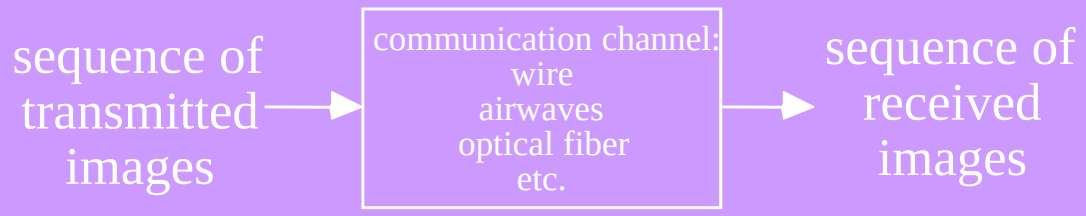
- Lossless Image Coding
- Lossy Image Coding
- JPEG Image Compression Standards
- Wavelet Compression

OBJECTIVES OF IMAGE COMPRESSION

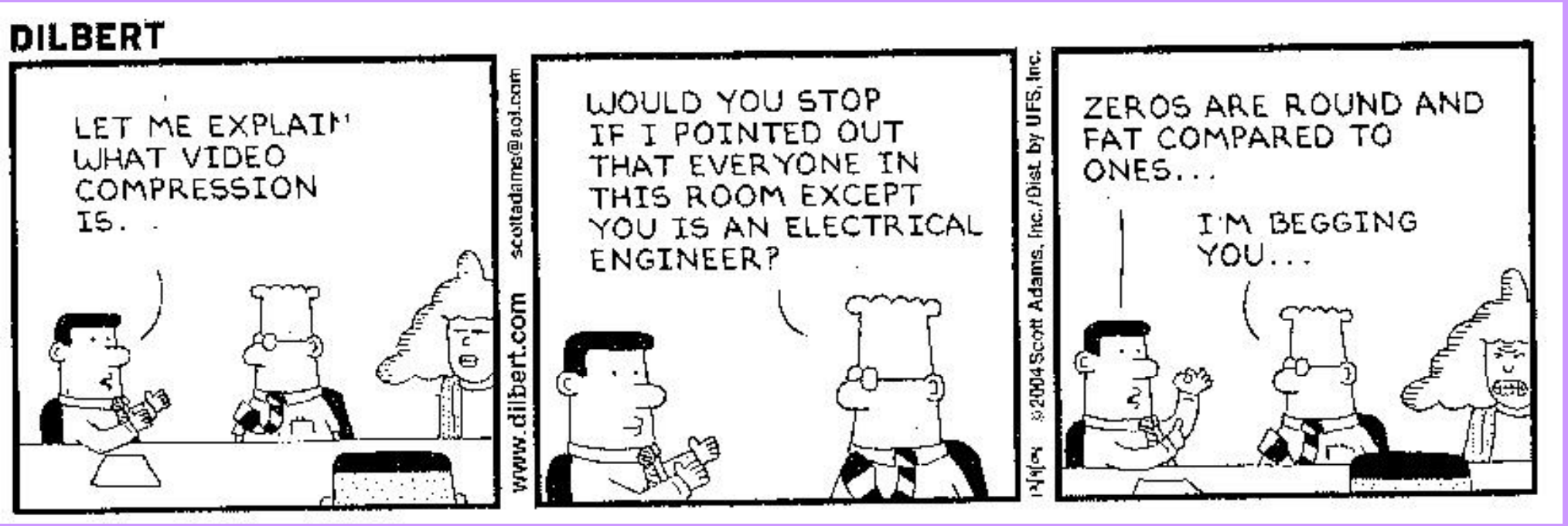
- Create a **compressed** image that "looks the same" (when **decompressed**) but can be stored in a fraction of the space.
- **Lossless compression** means the image can be **exactly** reconstructed.
- **Lossy compression** means that **visually redundant** information is removed. The decompressed image "looks" unchanged, but mathematically has lost information.
- **Image compression** is important for:
  - Reducing image **storage space**
  - Reducing image **transmission bandwidth**

Significance for Storage & Transmission





Compressed images can be sent at an increased rate: **More images per second !**


DILBERT
LET ME EXPLAIN WHAT VIDEO COMPRESSION IS. .
WOULD YOU STOP IF I POINTED OUT THAT EVERYONE IN THIS ROOM EXCEPT YOU IS AN ELECTRICAL ENGINEER?
ZEROS ARE ROUND AND FAT COMPARED TO ONES...
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Lossless Image Compression

Image Compression Measures

- **Bits Per Pixel (BPP)** is the **average** number of bits required to store the gray level of each pixel in **I**.

- **Uncompressed image** (K is gray scale range):

$$BPP(\mathbf{I}) = \log_2(K) = B$$

- Usually

$$B = \log_2(256) = 8.$$

Variable-Bit Codes

- The number of bits used to code pixels may **spatially vary**.
- Suppose that $\hat{\mathbf{I}}$ is a compressed version of **I**.
- Let B(i, j) = # of bits used to code pixel I(i, j). Then

$$\mathrm{BPP}(\hat{\mathbf{I}}) = \frac{1}{NM} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} B(i, j)$$

- If **total** number of bits in $\hat{\mathbf{I}}$ is B_{total}, then

$$\mathrm{BPP}(\hat{\mathbf{I}}) = \frac{1}{NM} B_{total}$$

Compression Ratio

- The **Compression Ratio (CR)** is

$$CR = \frac{BPP(\mathbf{I})}{BPP(\hat{\mathbf{I}})} > 1$$

- Both **BPP** and **CR** are used frequently.

LOSSLESS IMAGE COMPRESSION

- **Lossless** techniques achieve compression with **no loss of information**.
- The true image can be reconstructed **exactly** from the coded image.
- Lossless coding doesn't usually achieve **high compression** but has definite applications:
  - In **combination** with lossy compression, multiplying the gains
  - In applications where information loss is **unacceptable**.
- Lossless compression ratios **usually** in the range

$$\mathbf{2:1 \leq CR \leq 3:1}$$

but this may **vary** from image to image.

Methods for Lossless Coding

- Basically amounts to clever arrangement of the data.
- This can be done in many ways and in many domains (DFT, DCT, wavelet, etc)
- The most popular methods use **variable wordlength coding**.

Variable Wordlength Coding

(Huffman Code)

Variable Wordlength Coding (VWC)

- **Idea**: Use **variable wordlengths** to code gray levels.
- Assign **short wordlengths** to gray levels that occur **frequently** (redundant gray levels).
- Assign **long wordlengths** to gray levels that occur **infrequently**.
- On the average, the **BPP will be reduced**.

Image Histogram and VWC

- Recall the **image histogram** $H_{\mathbf{I}}$:

- Let B(k) = # of bits used to code gray-level k. Then

$$\mathrm{BPP}(\hat{\mathbf{I}}) = \frac{1}{NM}\sum_{k=0}^{K-1} B(k)H_{\mathbf{I}}(k)$$

- This is the **common measure of BPP for VWC**.

Image Entropy

- Recall the **normalized histogram** values

$$p_{\mathbf{I}}(k) = \frac{1}{NM} H_{\mathbf{I}}(k) \; ; \; k = 0,..., K\text{-}1$$

so $p_{\mathbf{I}}(k)$ = probability of gray level k

- The **entropy** of image **I** is then

$$E[\mathbf{I}] = -\sum_{k=0}^{K-1} p_{\mathbf{I}}(k) \log_2 p_{\mathbf{I}}(k)$$

Meaning of Entropy

- **Entropy** is a **measure of information** with nice properties for designing VWC algorithms.

- **Entropy** E[**I**] measures
  - **Complexity**
  - **Information Content**
  - **Randomness**

Maximum Entropy Image

- The entropy is **maximized** when

$$p_{\mathbf{I}}(k) = \frac{1}{K}\,;\; k = 0,\ldots,\; K\text{-}1$$

corresponding to a **flat histogram**. Then ($K = 2^B$)

$$E[\mathbf{I}] = -\sum_{k=0}^{K-1} \frac{1}{K} \log_2 \frac{1}{K} = B \cdot \sum_{k=0}^{K-1} \frac{1}{K} = B$$

- Generally, image entropy increases as the histogram is spread out.

Minimum Entropy Image

- The entropy is **minimized** when

$$p_{\mathbf{I}}(k_n) = 1 \text{ for some } k_n \; ; \; 0 \leq k_n \leq K\text{-}1$$

hence $p_{\mathbf{I}}(k_m) = 0$ for $m \neq n$.

- This is a **constant image** and $E[\mathbf{I}] = 0$.

- In fact it is always true that

$$0 \leq E[\mathbf{I}] \leq B$$

Significance of Entropy

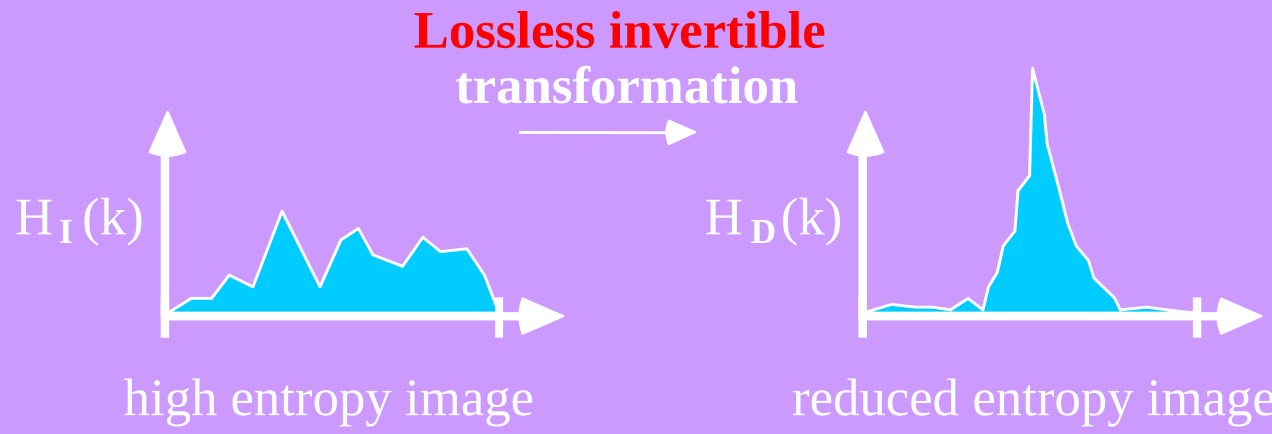
- An important **theorem** limits **how much** an image can be compressed by a lossless VWC:

$$\mathrm{BPP}(\hat{\mathbf{I}}) \geq \mathrm{E}[\mathbf{I}]$$

- A VWC is assumed **uniquely decodable**.
- The $p_{\mathbf{I}}(k)$ are assumed known at both transmitter **and** receiver.
- Thus an image with a flat histogram: $\mathrm{E}[\mathbf{I}] = B$ **cannot be compressed** by a VWC alone. Fortunately, we can often fix this situation by **entropy reduction**.
- Moreover, a **constant image** need not be sent: $\mathrm{E}[\mathbf{I}] = 0$!
- The entropy provides a compression **lower bound** as a **target**.

Image Entropy Reduction

- **Idea**: Compute a new image **D** with a more compressed histogram than **I** but with no loss of information.



- Must be able to recover **I exactly** from **D - no loss** of information.
- **Compressing** the histogram (Module 3) won't work, since information is lost when two gray levels k_1 and k_2 are mapped to the same new gray level k_3.

Entropy Reduction by Differencing (Simple DPCM)

- **Differential pulse-code modulation (DPCM)** is effective for lossless image entropy reduction.

- Images are mostly smooth: neighboring pixels **often** have similar values.

- Define a **difference image D** using either 1-D or 2-D differencing:

  (1-D) $D(i, j) = I(i, j) - I(i, j-1)$

  (2-D) $D(i, j) = I(i, j) - I(i-1, j) - I(i, j-1) + I(i-1, j-1)$

  for $0 \le i \le N-1$, $1 \le j \le M-1$.

- The new histogram $H_{\mathbf{D}}$ usually will be more compressed than $H_{\mathbf{I}}$, so that

$$E[\mathbf{D}] < E[\mathbf{I}]$$

- This is a rule of thumb for images, not a math result.

Reversing DPCM

- If (1) is used, then

$$I(i, j) = D(i, j) + I(i, j-1)$$

- The **first column** of **I** must also be transmitted.

- If (2) is used, then

$$I(i, j) = D(i, j) + I(i-1, j) + I(i, j-1) - I(i-1, j-1)$$

  where the first row and first column of **I** must also be transmitted.

- The overhead of the first row and column is small, but they can be **separately compressed**.

- **Hereafter** we will assume an image **I** has either been histogram compressed by DPCM or doesn't need to be.

Optimal Variable Wordlength Code

- **Recall** the theoretical **lower bound** using a VWC:

$$\mathrm{BPP}(\hat{\mathbf{I}}) \geq \mathrm{E}[\mathbf{I}] = -\sum_{k=0}^{K-1} p_{\mathbf{I}}(k) \log_2 p_{\mathbf{I}}(k)$$

- Observe: In any VWC, coding each gray-level k using **wordlength** L(k) bits gives an **average wordlength**:

$$\mathrm{BPP}(\hat{\mathbf{I}}) = \sum_{k=0}^{K-1} p_{\mathbf{I}}(k) L(k)$$

- Compare with the above. If $L(k) = -\log_2 p_{\mathbf{I}}(k)$ then an optimum code has been found - lower bound attained!

Optimal VWC

- **IF** we can find such a VWC such that

  $$L(k) = -\log_2 p_{\mathbf{I}}(k) \text{ for } k = 0, \ldots, K-1$$

  then the code is **optimal**.

- It is impossible if $-\log_2 p_{\mathbf{I}}(k) \neq$ an integer for **some** k.

- So: define an **optimal code** $\hat{\mathbf{I}}$ as one that satisfies:
  - $\mathrm{BPP}(\hat{\mathbf{I}}) \leq \mathrm{BPP}(\hat{\mathbf{I}}')$ of any other code $\hat{\mathbf{I}}'$
  - $\mathrm{BPP}(\hat{\mathbf{I}}) = E[\mathbf{I}]$ if $[-\log_2 p_{\mathbf{I}}(k)] =$ integers

The Huffman Code

- The **Huffman algorithm** yields an **optimum code**.
- For a set of gray levels {0 ,.., K-1} it gives a set of **code words** c(k); $0 \leq k \leq K\text{-}1$ such that

$$BPP(\hat{\mathbf{I}}) = \sum_{k=0}^{K-1} p_{\mathbf{I}}(k)\, L\left[c(k)\right]$$

is the **smallest possible**.

Huffman Algorithm

- Form a **binary tree** with branches labeled by the gray-levels k_m and their probabilities $p_I(k_m)$:

  (0) Eliminate any k_m where $p_I(k_m) = 0$.
  (1) Find 2 **smallest probabilities** $p_m = p_I(k_m)$, $p_n = p_I(k_n)$.
  (2) Replace by $p_{mn} = p_m + p_n$ to **form a node**; reduce list by 1
  (3) Label the branch for k_m with (e.g.) '1' and for k_n with '0'.
  (4) Until list has only 1 element (root reached), return to (1).

- In step (3), values '1' and '0' are assigned to element pairs (k_m, k_n), elements triples, etc. as the process progresses.

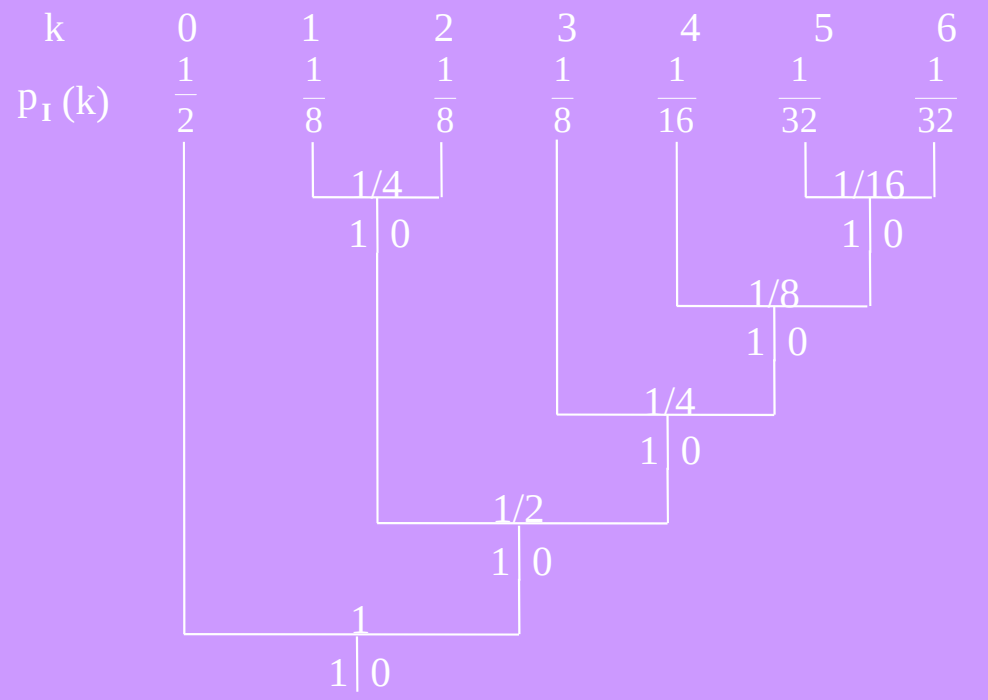
Huffman Tree Example 1

- There are K = 8 values {0 ,.., 7} to be assigned codewords:

$p_I(0) = 1/2$	$p_I(4) = 1/16$
$p_I(1) = 1/8$	$p_I(5) = 1/32$
$p_I(2) = 1/8$	$p_I(6) = 1/32$
$p_I(3) = 1/8$	$p_I(7) = 0$

- The process creates a **binary tree**, with values '1' and '0' placed (e.g.) on the right and left branches at each stage:

Huffman Tree Example 1



c(k)	1	011	010	001	0001	00001	00000
L[c(k)]	1	3	3	3	4	5	5

BPP = Entropy = 2.1875 bits

CR = 1.37 : 1

Huffman Tree Example 2

- There are K = 8 values {0 ,.., 7} to be assigned codewords:

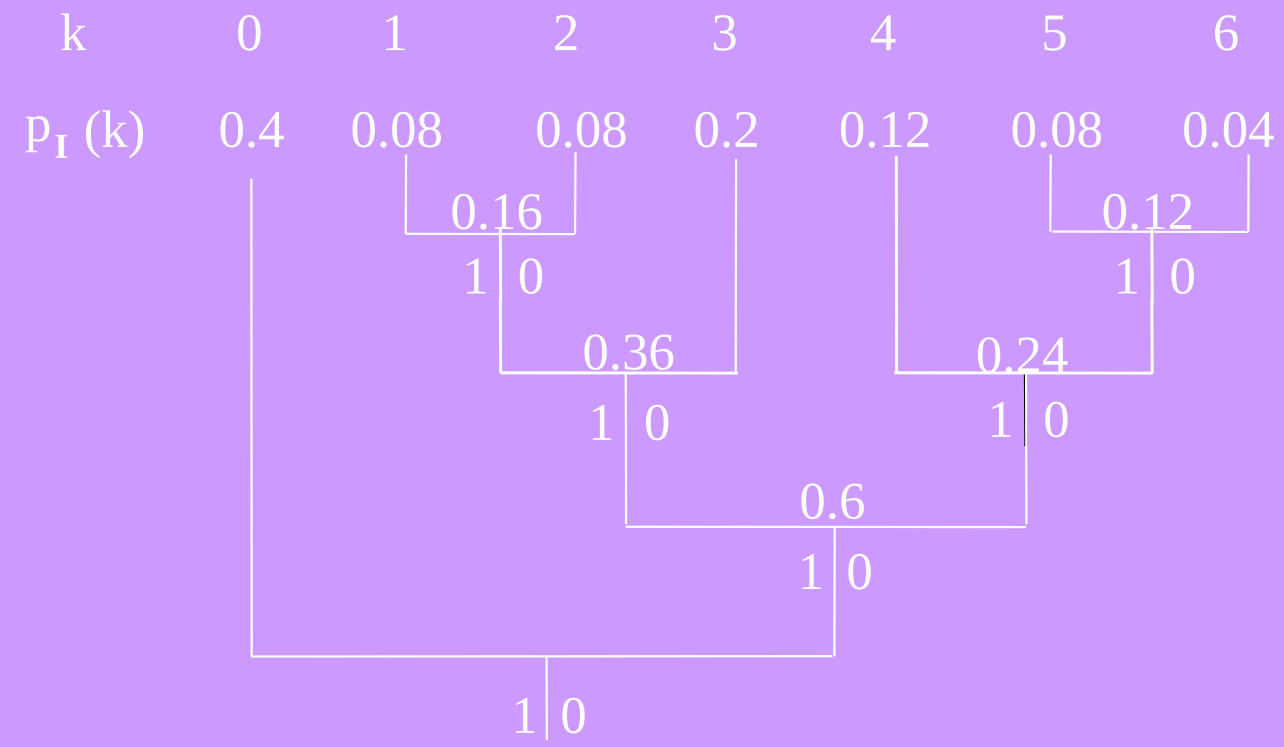
$p_I(0) = 0.4$ $p_I(4) = 0.12$

$p_I(1) = 0.08$ $p_I(5) = 0.08$

$p_I(2) = 0.08$ $p_I(6) = 0.04$

$p_I(3) = 0.2$ $p_I(7) = 0.00$

Huffman Tree Example 2



k	0	1	2	3	4	5	6
p_I (k)	0.4	0.08	0.08	0.2	0.12	0.08	0.04
c(k)	1	0111	0110	010	001	0001	0000
L[c(k)]	1	4	4	3	3	4	4

BPP = 2.48 bits
Entropy = 2.42 bits
CR = 1.2 : 1

Huffman Decoding

- The Huffman code is a **uniquely decodable** code. There is **only one interpretation** for a series of codewords (series of bits).

- Decoding progresses by **traversing the tree**.

Huffman Decoding Example

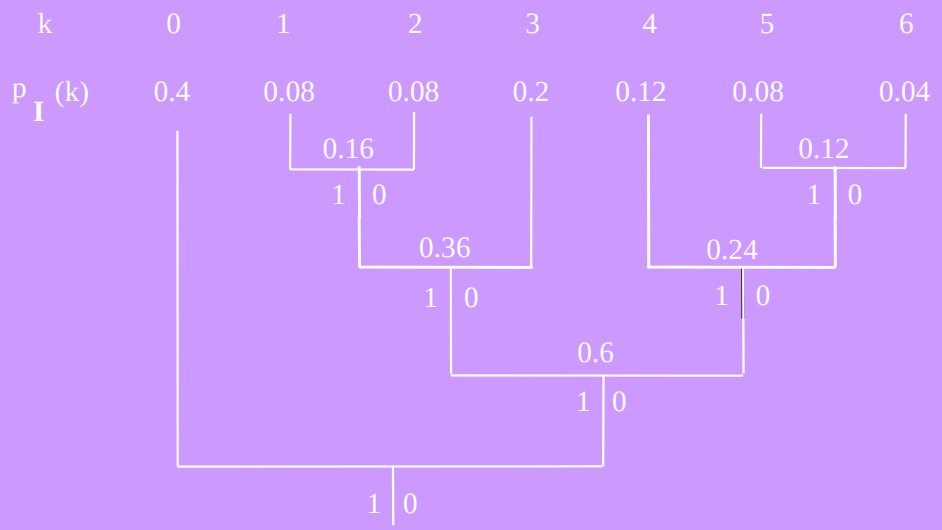
- In the **second example**, this sequence is received:

  0001011010111000001000010001011011101 0

- It is sequentially examined until a codeword is identified. This continues until all are identified:

  0001 0110 1 0111 0000 010 0001 0001 0110 1 1 1 010

- The decoded sequence is: 5 2 0 1 6 3 5 5 2 0 0 0 3



Comments on Huffman Coding

- **Huffman image compression** usually
$$2{:}1 \leq CR \leq 3{:}1$$

- Huffman codes are quite **noise-sensitive** (much more so than an uncompressed image).

- **Error-correction coding** can improve this but increases increase the coding rate somewhat.

- Some Huffman codes aren't computed from the image statistics - instead assume "typical" frequencies of occurrence of values (gray-level or transform values) – as in JPEG.

Lossy Image Compression

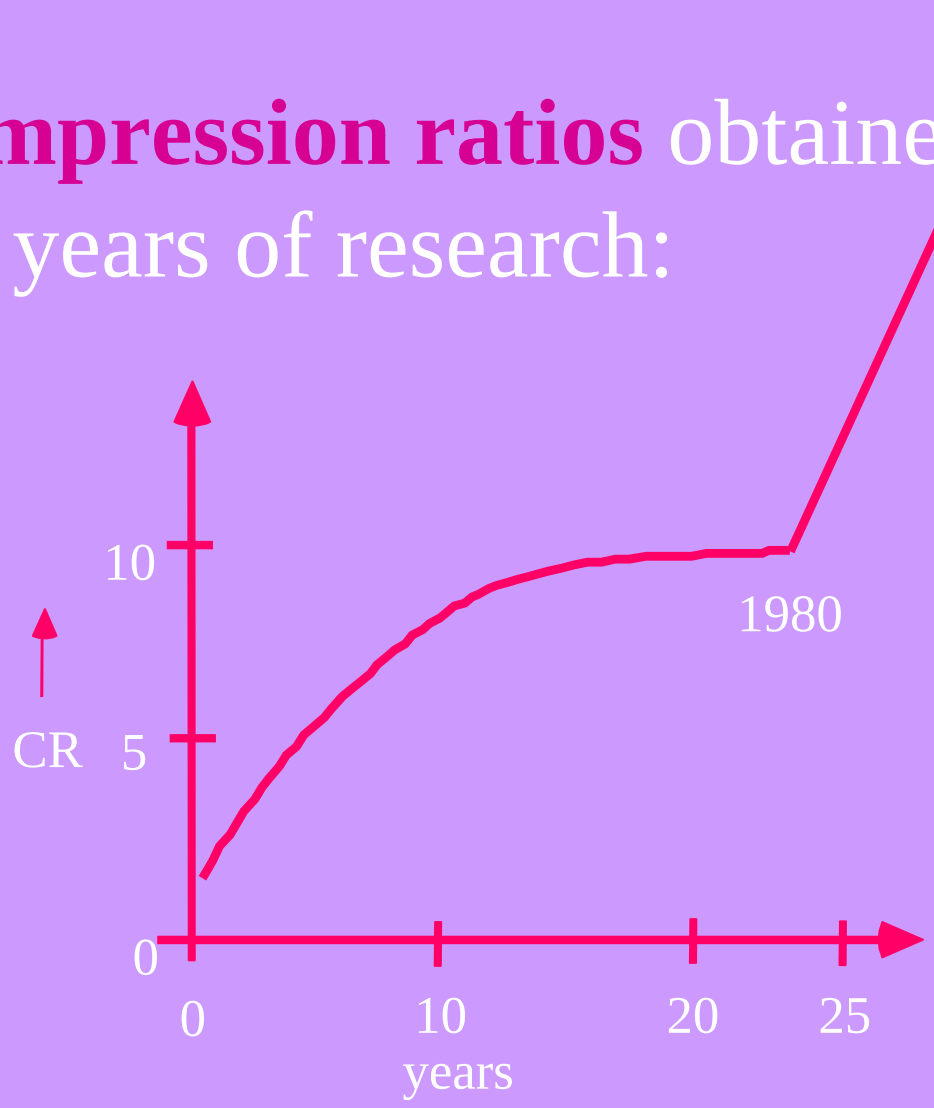
LOSSY IMAGE CODING

- **Many** approaches proposed.

- We will review a **few** popular approaches.
  - **Block Truncation Coding** (BTC) – simple, fast
  - **Vector Quantization Coding** (VQC) - costly but high-quality results
  - **Discrete Cosine Transform** (DCT) Coding and the **JPEG Standard**
  - **Wavelet Image Coding** and **JPEG-2000**

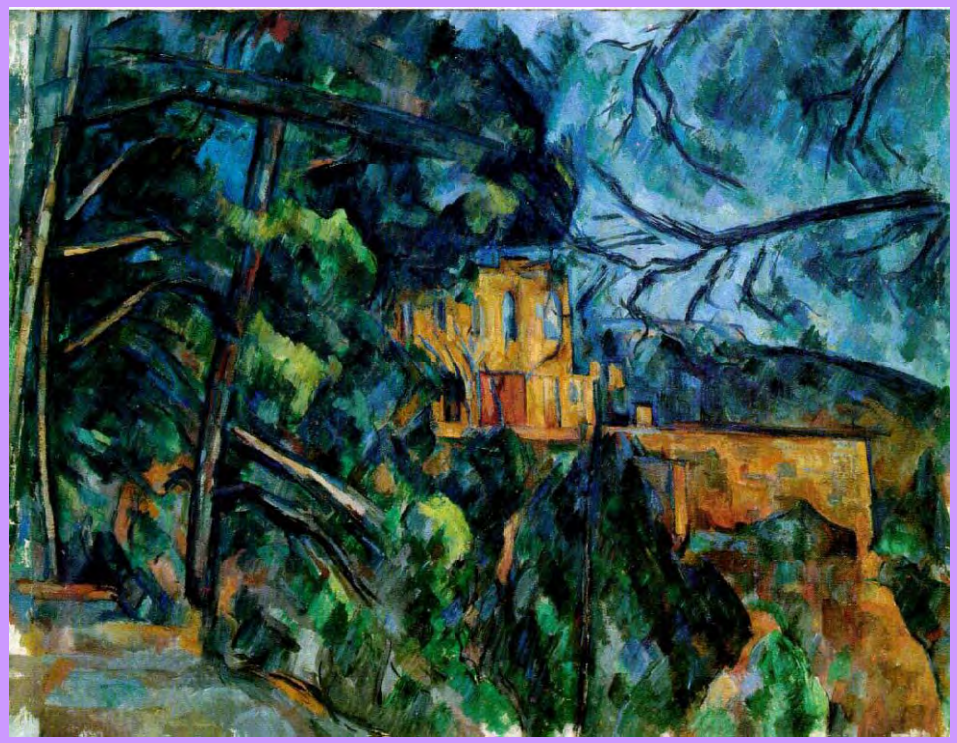
History of Image Compression Advances

- The **compression ratios** obtained over the first 30 years of research:



Goals of Lossy Coding

- To optimize and balance the three C's
  - **Compression** achieved by coding
  - **Computation** required by coding and decoding
  - **Cuality** of the decompressed image



Artistic image coding with significant loss
(Cezanne)

Artistic image coding with little loss
(Degas)

Broad Methodology of Lossy Compression

- There have been **many** proposed lossy compression methods.

- The successful broadly (and loosely) follow three steps.

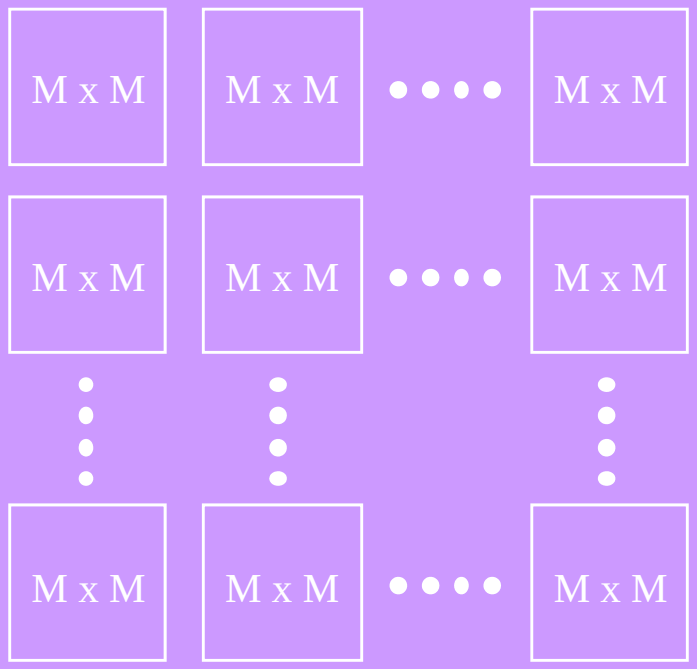
  (1) Transform the image to another domain and/or extract specific features.

  (2) Quantize in this domain or those features.

  (3) Efficiently organize and/or entropy code the quantized data.

Block Coding of Images

- **Most** lossy methods begin by partitioning the image into **sub-blocks** that are individually coded. **Wavelet methods** are an **exception**.



Why Block Coding?

- **Reason:** images are **highly nonstationary: different areas** of an image may have **different properties**, e.g., more high or low frequencies, more or less detail etc.

- Thus, **local coding** is **more efficient**. Wavelet methods provide localization without blocks.

- Typical block sizes: **4 x 4**, **8 x 8**, **16 x 16**.

Block Truncation Coding (BTC)

Block Truncation Coding (BTC)

- **Fast**, but limited compression.
- Uses **4 x 4 blocks** each containing 16 · 8 = **128 bits**.
- Each 4 x 4 block is coded identically so no need to **index them**.
- Consider the coding of a **single block** which we will denote {I(1) ,..., I(16)}.

BTC Coding Algorithm

(1) Quantize **block sample mean:**

$$\bar{I} = \frac{1}{16}\sum_{p=1}^{16} I(p)$$

and transmit/store it with B_1 bits.

(2) Quantize **block sample standard deviation:**

$$\bar{\sigma}_I = \sqrt{\frac{1}{16}\sum_{p=1}^{16}\left[I(p) - \bar{I}\right]^2}$$

and transmit/store it with B_2 bits.

BTC Coding Algorithm

- Compute a **16-bit binary block:**

$$b(p) = \begin{cases} 1 \; ; & \text{if } I(p) \geq \bar{I} \\ 0 \; ; & \text{if } I(p) < \bar{I} \end{cases}$$

121	114	56	47
37	200	247	255
16	0	12	169
43	5	7	251

$\bar{I} = 98.75$ →

1	1	0	0
0	1	1	1
0	0	0	1
0	0	0	1

which requires 16 bits to transmit/store.

BTC Quantization

- The quality/compression of BTC-coded images depends on the **quantization** of mean and standard deviation.

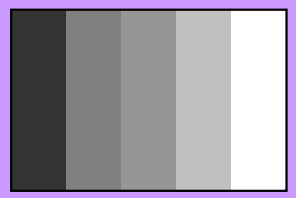
- If $B_1 = B_2 = 8$, 32 bits / block are transmitted:

  CR = 128 / 32 = 4:1

- OK quality if $B_1 = 6$, $B_2 = 4$ (26 bits total):

  CR = 128 / 26 ≈ 5:1

- Using $B_1 > B_2$ is OK: the eye is quite sensitive to the **presence** of variation, but **not** to the **magnitude** of variation:



Mach Band Illusion

BTC Block Decoding

- To form the "decoded" pixels J(1) ,...., J(16):

  (1) Let Q = number of '1's, P = number of '0's in binary block.

  (2) If $\begin{cases} b(p)=1 \;\; ; \;\; \text{set } J(p) = \bar{I} + \bar{\sigma}_I/A \\ b(p)=0 \;\; ; \;\; \text{set } J(p) = \bar{I} - \bar{\sigma}_I \cdot A \end{cases}$

  where

$$A = \sqrt{\frac{Q}{P}}$$

- It is possible to show that this forces $\bar{J} \approx \bar{I}$ and $\bar{\sigma}_J \approx \bar{\sigma}_I$

BTC Block Decoding Example

- In our example: Q = 7, P = 9, A = 0.8819

$$\bar{I} = \mathrm{INT}\left[98.75 + 0.5\right] = 99$$

$$\bar{\sigma}_I = \mathrm{INT}\left[92.95 + 0.5\right] = 93$$

- If $\begin{cases} b(p)=1 \;;\; \text{set } J(p) = 99 + \mathrm{INT}\left[93/0.882+0.5\right] = 204 \\ b(p)=0 \;;\; \text{set } J(p) = 99 - \mathrm{INT}\left[93 \cdot 0.882+0.5\right] = 17 \end{cases}$

204	204	17	17
17	204	204	204
17	17	17	204
17	17	17	204

DEMO

Here $\bar{J} \approx 98.8$

$\bar{\sigma}_J \approx 77.3$

Comments on BTC

- Attainable compressions by simple BTC in the range 4:1 – 5:1.
- Combining with **entropy coding** of the quantized data, in the range 10:1.
- Popular in **low-bandwidth, low-complexity** applications. Used by the Mars Pathfinder!

Vector Quantization Image Compression

Vector Quantization Image Coding

- A very light overview!
- Actually, there are **many variations** of this technique.
- **Coding** via VQ is **very computation-intensive**. **Decoding** is not.
- VQ algorithms also (usually) use **4 x 4 blocks** – each of which is coded identically.

VQ Codebook

- The image **I** is broken into 4 x 4 blocks

  $\mathbf{b}_m$; m = 1 ,…, MN/16.

- From these, compute a **codebook** of P "typical" 4 x 4 blocks $\mathbf{t}_p$, p = 1 ,..., P.

- This is done using a **clustering algorithm** which divides a space of points into likely groups according to a distance criterion.

- Usually $256 \leq P \leq 1024$.

- Simplest: Use K-Means Clustering with K=P.

K-Means: Simple Clustering Method

- Given MN/16 **16-D** 4 x 4 blocks $\mathbf{b}_m$; m = 1 ,…, MN/16.

- **Partition $\mathbf{b}_m$ into K 16-D clusters C** = $\{C_1, C_2, \ldots, C_K\}$ by minimizing the **within-cluster sum-of-squares:**

$$\arg\min_{\mathbf{C}} \sum_{k=1}^{K} \sum_{\mathbf{b}_j \in C_k} \left\| \mathbf{b}_j - \bar{\mathbf{b}}_k \right\|$$

  where the $\bar{\mathbf{b}}_k$ are the **block means** (centroids).

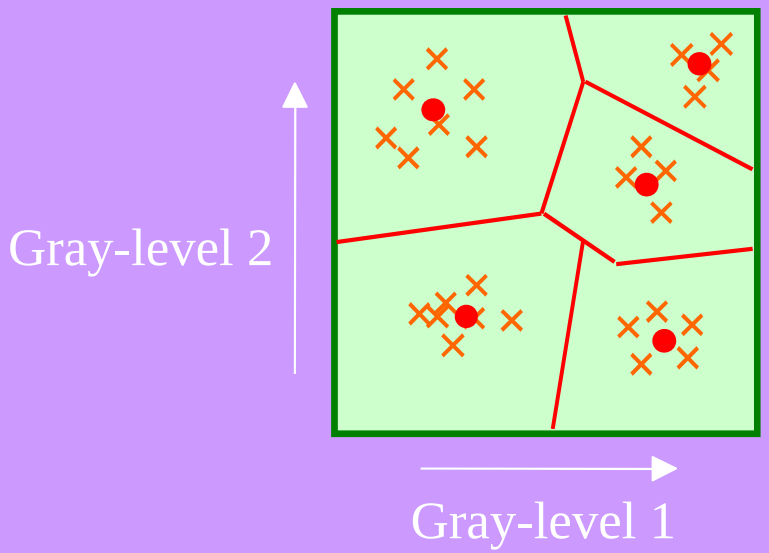
- **Lloyd's Algorithm:** Select initial means $\left\{\bar{\mathbf{b}}_1^{(0)}, \bar{\mathbf{b}}_2^{(0)}, \ldots, \bar{\mathbf{b}}_K^{(0)}\right\}$ (e.g. randomly from the $\mathbf{b}_m$). Then iterate between two steps:
  (1) Assign $\mathbf{b}_m$; m = 1 ,…, MN/16 to cluster q with mean $\bar{\mathbf{b}}_q^{(0)}$ closest to $\mathbf{b}_m$.
  (2) Calculate new cluster means

$$\bar{\mathbf{b}}_k^{(r+1)} = \frac{1}{\mathrm{card}\left[C_k^{(r)}\right]} \sum_{\mathbf{b}_j \in C_k^{(r)}} \mathbf{b}_j$$

- **Complexity:** $O\left[\left(\frac{MN}{16}\right)^{16P+1} \log\left(\frac{MN}{16}\right)\right]$

2 x 1 Codebook Example

- If 2 x 1 blocks were used, the space might look like this:



- The cluster **centroids** are the coordinates of the "typical" blocks!

- In VQ image compression, the space is **16-D**, rather than 2-D.

Codebook Computation

- Computation of the codebook directly from the image is **very time-consuming.**
- **Unrealistic for real-time encoding.**
- **Approximate** (sub-optimal) algorithms exist which are faster – e.g., Equitz Algorithm.
- A **"universal" codebook** can be used in applications where the images are always similar. Compute the codebook once from many images, then don't change it.

Codebook Search

- If codebook is computed from **I**, finding the centroid of the cluster that block **b** belongs to is **simple**.
- If a universal codebook is used, then the closest centroid must be **searched for**.
- The typical block $\mathbf{t}_{q^*}$ is found that is closest to **b**:

$$q^* = \arg \min_q \sqrt{\frac{1}{16} \sum_{p=1}^{16} \left[b(p) - t_q(p) \right]^2}$$

- The index q* is the **code** for **b**! This is what is stored or transmitted.

VQ Decoding

- While codebook construction and search are very time consuming ….
- Decoding is **very fast**! Just a codebook look-up!
- Of course the codebook is **known by the decoder** (communications scenario).
- Given a code (index) q for block **b**, the typical code $\mathbf{t}_q$ represents **b** in the **decompressed image**.

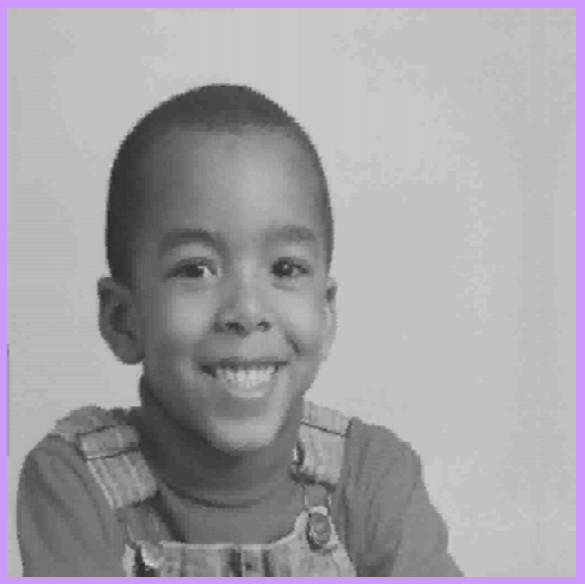
VQ Comments

- VQ is an excellent example of an **unbalanced codec** (coder-decoder).
- Great for apps where coding time is less important but **decoding time is very important**: e.g., image libraries.
- Compression ratios CR = 15:1 - 20:1 are **typical**.

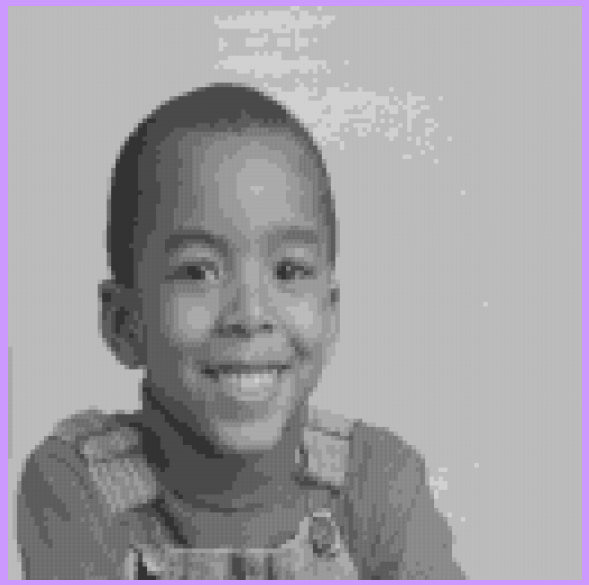
Simple VQ Example 1

Original *Kevin*

16 : 1



32 : 1



64 : 1

Simple VQ Example 2

Original *Steven*

16 : 1

32 : 1

64 : 1

Classified VQ

- One effective variation is **classified VQ** where the block means and standard deviations are computed, and the blocks normalized by them:

$$\beta(p) = \frac{b(p) - \bar{b}}{\bar{\sigma}_b}$$

- The moments $\bar{b}$ and $\bar{\sigma}_b$ are quantized and stored/transmitted **separately**.

- VQ is applied to the normalized blocks β to form a "classified codebook" which can be much smaller or more accurate.

JPEG

JPEG

- The **JPEG Standard** remains the most widely-used method of compressing images.
- JPEG = **Joint Photographic Experts Group**, an international standardization committee.
- Standardization allows device and software manufacturers to have an **agreed-upon format** for compressed images.
- In this way, everyone can “talk” and “share” images.
- The overall JPEG Standard is quite complex, but the core algorithm is based on the **Discrete Cosine Transform (DCT).**

Discrete Cosine Transform (DCT)

- The **DCT** of an N x M image or sub-image:

$$\tilde{\tilde{I}}(u, v) = \frac{4C_N(u)C_M(v)}{NM} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} I(i, j) \cos\left[\frac{(2i+1)u\pi}{2N}\right] \cos\left[\frac{(2j+1)v\pi}{2M}\right]$$

- The **IDCT**:

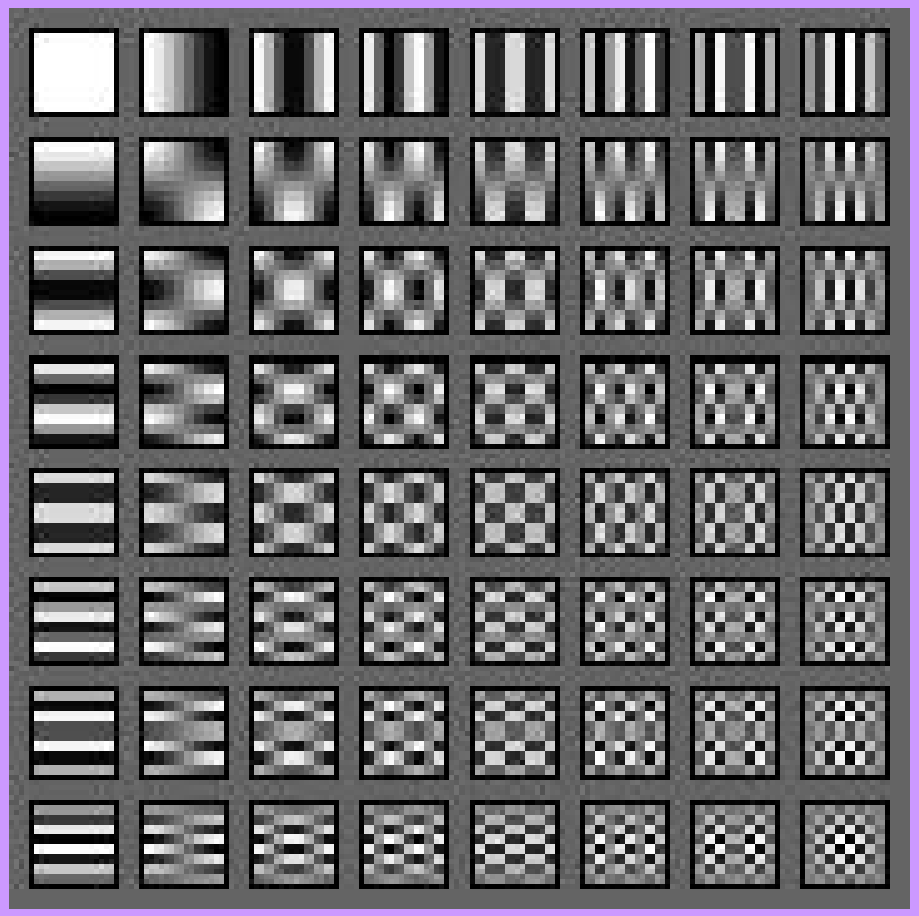
$$I(i, j) = \sum_{u=0}^{N-1} \sum_{v=0}^{M-1} C_N(u)C_M(v)\, \tilde{\tilde{I}}(u, v) \cos\left[\frac{(2i+1)u\pi}{2N}\right] \cos\left[\frac{(2j+1)v\pi}{2M}\right]$$

where

$$C_N(u) = \begin{cases} 1/\sqrt{2} & ; u = 0 \\ 1 & ; u = 1, \ldots, N-1 \end{cases}$$

DCT Basis Functions

- Displayed as 8 x 8 images:

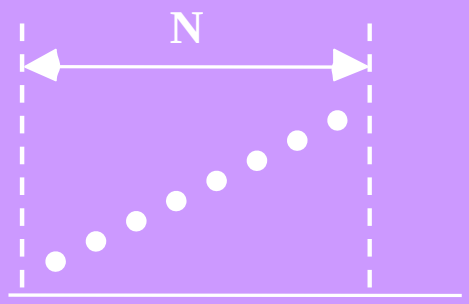


DCT vs. DFT

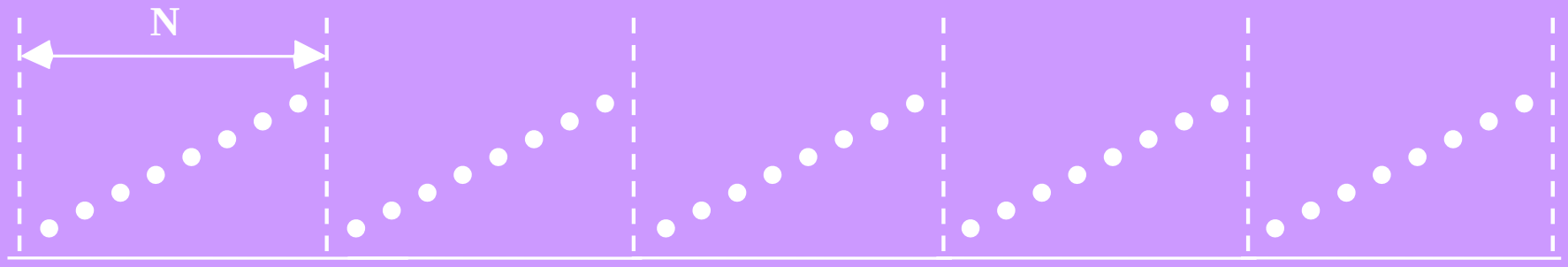
- JPEG is based on quantizing and re-organizing block DCT coefficients.
- The DCT is very similar to the DFT. **Why use the DCT?**
- First, $O(N^2 \log N^2)$ algorithms exist for DCT - slightly faster than DFT: all real, **integer-only** arithmetic.
- The DCT yields **better-quality compressed images** than the DFT, which suffers from more serious **block artifacts**.
- **Reason**: The DFT implies the image is N-periodic, whereas the DCT implies the **once-reflected image** is 2N-periodic.

Periodicity Implied by DFT

- A length-N (1-D) signal:

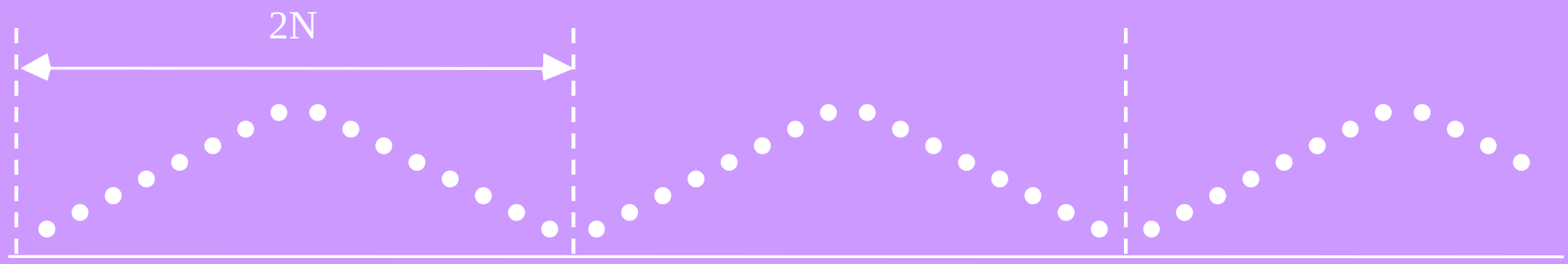


- **Periodic extension** implied by the DFT:



Periodicity Implied by DCT

- Periodic extension of **reflected signal** implied by the DCT:



- **In fact** (good exercise): The DFT of the length-2N reflected signal yields (two periods of) the DCT of the length-N signal.

Quality Advantage of DCT Over DFT

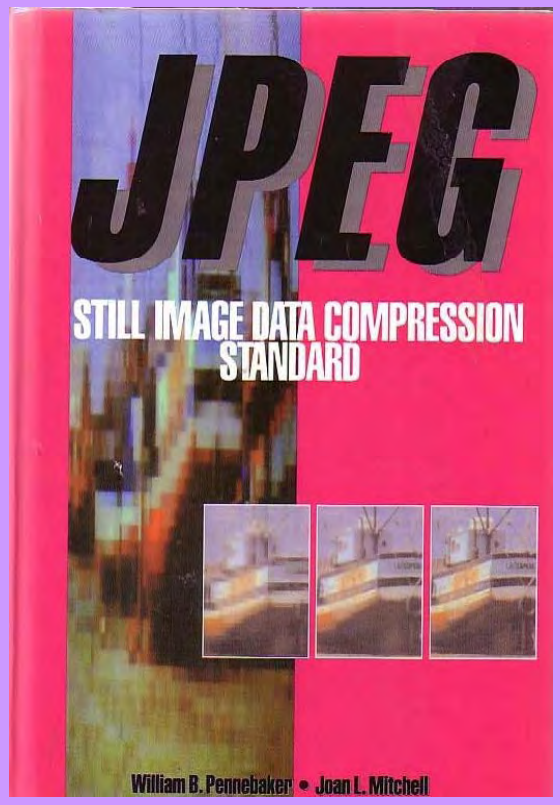
- The periodic extension of the DFT contains **high frequency discontinuities**.
- These high frequencies **must be well-represented** in the code, or the edge of the blocks will degrade, creating visible **blocking artifacts**.
- The DCT does **NOT** have meaningless discontinuities to represent - so less to encode, and so, **higher quality for a given CR.**

Overview of JPEG

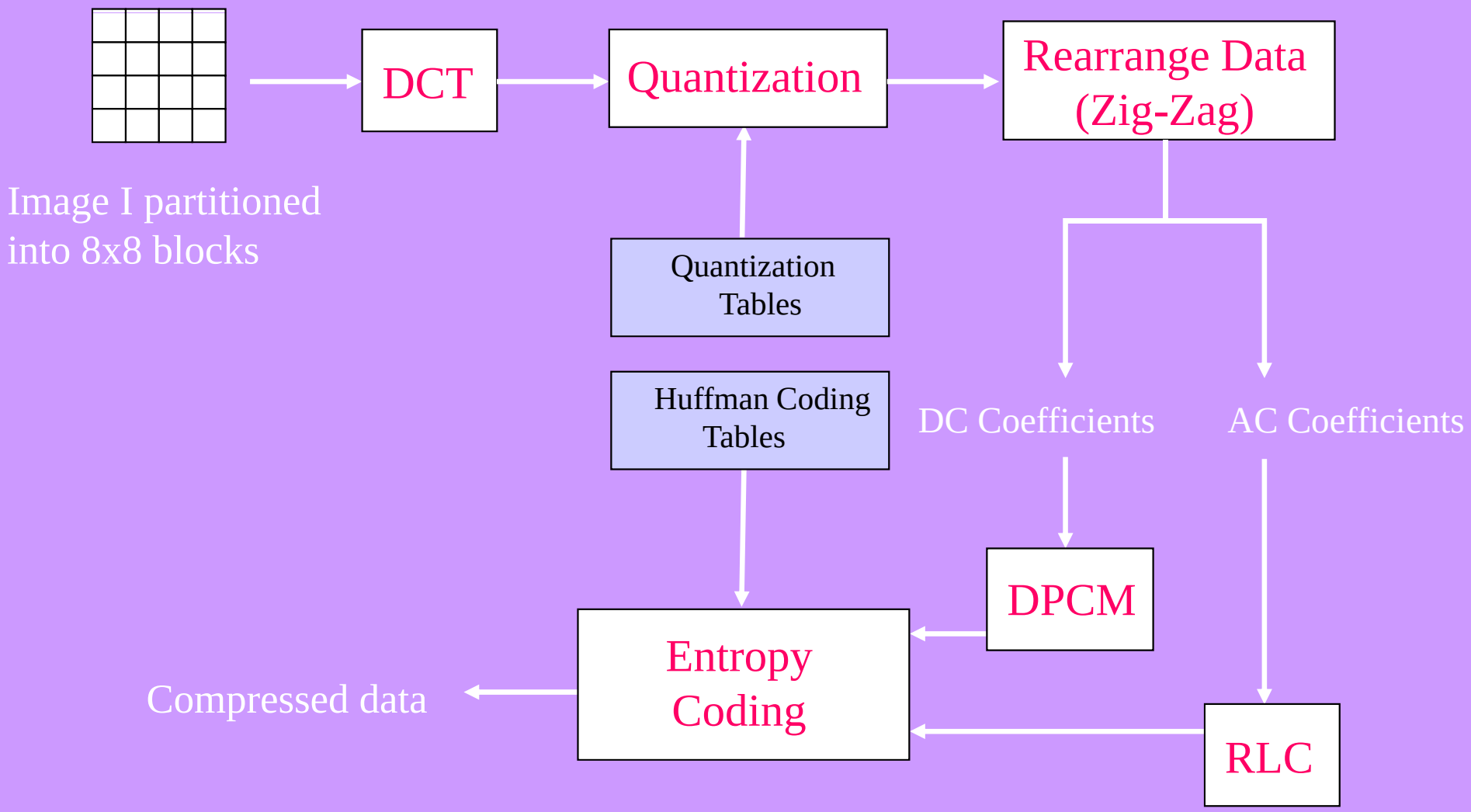
- The commercial industry standard - formulated by the CCIT **Joint Photographic Experts Group** (JPEG).
- Uses the **DCT** as the central transform.
- Overall JPEG algorithm is **quite complex**. Three components:
  - **JPEG baseline system** (basic lossy coding)
  - JPEG extended features (12-bit; progressive; arithmetic)
  - JPEG lossless coder

JPEG Standard Documentation

- The written standard is **gigantic**. Best resource (Pennebaker and Mitchell):



JPEG Baseline Flow Diagram



JPEG Baseline Algorithm

(1) Partition image into **8 x 8 blocks**, transform each block using the DCT. Denote these blocks by: $\tilde{\tilde{I}}_k(u, v)$

(2) Pointwise divide each block by an 8 x 8 **user-defined normalization array** Q(u, v). This is **stored / transmitted** as part of the code. Usually designed using sensitivity properties of **human vision**.

(3) The JPEG committee performed a human study on the **visibility of DCT basis functions** under a specific viewing model.

JPEG Baseline Algorithm

(3) **Uniformly quantize** the result

$$\tilde{\tilde{I}}_k^Q(u, v) = \mathrm{INT}\left[\frac{\tilde{\tilde{I}}_k(u, v)}{Q(u, v)} + 0.5\right]$$

yielding an **integer array** with **many zeros**.

JPEG Quantization Example

- A block DCT (via integer-only algorithm)

$$\tilde{\tilde{\mathbf{I}}}_k = \begin{bmatrix} 1260 & -1 & -12 & -5 & 2 & -2 & -3 & 1 \\ -23 & -17 & -6 & -3 & -3 & 0 & 0 & -1 \\ -11 & -9 & -2 & 2 & 0 & -1 & -1 & 0 \\ -7 & -2 & 0 & 1 & 1 & 0 & 0 & 0 \\ -1 & -1 & 1 & 2 & 0 & -1 & 1 & 1 \\ 2 & 0 & 2 & 0 & -1 & 1 & 1 & -1 \\ -1 & 0 & 0 & -1 & 0 & 2 & 1 & -1 \\ -3 & 2 & -4 & -2 & 2 & 1 & -1 & 0 \end{bmatrix}$$

- Typical JPEG normalization array:

$$\mathbf{Q} = \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$

JPEG Quantization Example

- The resulting quantized DCT array:

$$\tilde{\mathbf{I}}_k^Q = \begin{bmatrix} 79 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- Notice all the **zeros.** The **DC value** $\tilde{\tilde{I}}_k^Q(0, 0)$ is in the upper left corner.
- This works because the DCT has **good energy compaction,** leading to a sparse image representation.

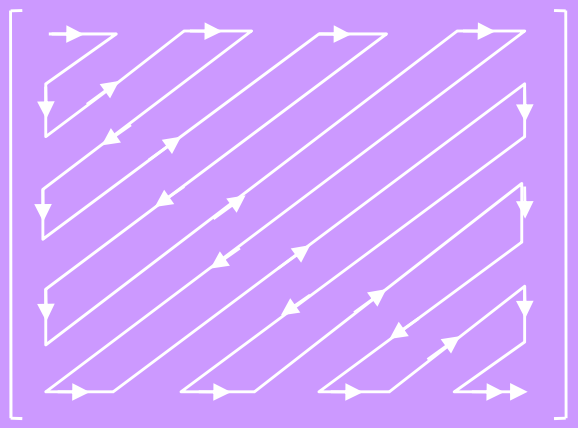
Data Re-Arrangement

- Rearrange quantized AC values

$$\tilde{\tilde{I}}_k^Q(u, v) \; ; (u, v) \neq (0, 0)$$

- This array contains **mostly zeros,** especially at high frequencies. So, rearrange the array into a 1-D vector using **zig-zag ordering:**

$$\begin{bmatrix} 0 & 1 & 5 & 6 & 14 & 15 & 27 & 28 \\ 2 & 4 & 7 & 13 & 16 & 26 & 29 & 42 \\ 3 & 8 & 12 & 17 & 25 & 39 & 41 & 43 \\ 9 & 11 & 18 & 24 & 31 & 40 & 44 & 53 \\ 10 & 19 & 23 & 32 & 39 & 45 & 52 & 54 \\ 20 & 22 & 33 & 38 & 46 & 51 & 55 & 60 \\ 21 & 34 & 37 & 47 & 50 & 56 & 59 & 61 \\ 35 & 36 & 48 & 49 & 57 & 58 & 62 & 63 \end{bmatrix}$$



Data Re-Arrangement Example

- Reordered quantized block from the previous example:

[79 0 -2 -1 -1 -1 0 0 -1 (55 0's)]

- **Many** zeros.

Handling of DC Coefficients

- Apply **simple DPCM** to the **DC values** $\tilde{\tilde{I}}_k^Q(0, 0)$ between adjacent blocks to **reduce** the **DC entropy.**

- The **difference** between the current-block DC value and the left-adjacent-block DC value is found:

$$e(k) = \tilde{\tilde{I}}_k^Q(0, 0) - \tilde{\tilde{I}}_{k-1}^Q(0, 0)$$

- The differences e(k) are losslessly coded by a **lossless** JPEG **Huffman coder** (with agreed-upon table).

- The first column of DC values must be retained to allow reconstruction.

Huffman Coding of DC Differences

SSSS	DPCM difference
0	0
1	−1,1
2	−3,−2,2,3
3	−7,...,−4,4,...,7
4	−15,...,−8,8,...,15
5	−31,...,−16,16,...,31
6	−63,...,−32,32,...,63
7	−127,...,−64,64,...,127
8	−255,...,−128,128,...,255
9	−511,...,−256,256,...,511
10	−1023,...,−512,512,...,1023
11	−2047,...,−1024,1024,...,2047
12	−4095,...,−2048,2048,...,4095
13	−8191,...,−4096,4096,...,8191
14	−16383,...,−8192,8192,...,16383
15	−32767,...,−16384,16384,...,32767
16	32768

- A Huffman Code is generated and stored/sent indicating which **category** (SSSS) the DC value falls in (See Pennebaker and Mitchell for the Huffman codes).
- SSSS is also the # bits allocated to code the DPCM differences and sign

Run-Length Coding of AC Coefficients

- The AC vector contains **many** zeros.
- By using RLC considerable compression is attained.
- The AC vector is converted into 2-tuples (Skip, Value), where

  Skip = number of zeros preceding a non-zero value

  Value = the following non-zero value.

- When final non-zero value encountered, send (0,0) as **end-of-block**.

Huffman Coding of AC Values

SSSS	AC coefficients
1	−1,1
2	−3,−2,2,3
3	−7,....,−4,4,....,7
4	−15,....,−8,8,....,15
5	−31,....,−16,16,....,31
6	−63,....,−32,32,....,63
7	−127,....,−64,64,....,127
8	−255,....,−128,128,....,255
9	−511,....,−256,256,....,511
10	−1023,....,−512,512,....,1023
11	−2047,....,−1024,1024,....,2047
12	−4095,....,−2048,2048,....,4095
13	−8191,....,−4096,4096,....,8191
14	−16383,....,−8192,8192,....,16383
15	−32767,....,−16384,16384,....,32767

- The AC pairs (Skip, Value) are coded using **another** JPEG Huffman coder.
- A Huffman code represents which category (SSSS) the AC pair falls in.
- RRRR bits are used to represent the runlength of zeros, SSSS additional bits are used to represent the AC magnitude and sign.
- See Pennebaker and Mitchell for details!

JPEG Decoding

- **Decoding** is accomplished by reversing the Huffman coding, RLC and DPCM coding to recreate $\tilde{\mathbf{I}}_k^Q$
- Then **multiply by the normalization array** to create the lossy DCT

$$\tilde{\mathbf{I}}_k^{lossy} = \mathbf{Q} \otimes \tilde{\mathbf{I}}_k^Q$$

- The decoded image block is the IDCT of the result

$$\mathbf{I}_k^{lossy} = \text{IDCT}\left[\tilde{\mathbf{I}}_k^{lossy}\right]$$

JPEG Decoding

- The **overall compressed image** is recreated by putting together the compressed 8 x 8 pieces:

$$\mathbf{I}^{\text{lossy}} = \left[\mathbf{I}_{k}^{\text{lossy}} \right]$$

- The compressions that can be attained range over:
  - 8 : 1 (very high quality)
  - 16 : 1 (good quality)
  - 32 : 1 (poor quality for most applications)

JPEG Examples

Original
512 x 512
Barbara

16 : 1

32 : 1

64 : 1

Original
512 x 512
Boats

16 : 1

32 : 1

64 : 1

DEMO

COLOR JPEG

- Basically, RGB image is converted to YUV or YCrCb image (Y = intensity, UV or CrCb = chrominances – see Module 10)
- Each channel Y, U, V is **separately** JPEG-coded (no cross-channel coding)
- Chrominance images may be **subsampled** first
- Different quantization / normalization table may be used

JPEG Color Quantization Table

$$Q = \begin{bmatrix} 17 & 18 & 24 & 47 & 99 & 99 & 99 & 99 \\ 18 & 21 & 26 & 66 & 99 & 99 & 99 & 99 \\ 24 & 26 & 56 & 99 & 99 & 99 & 99 & 99 \\ 47 & 66 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \\ 99 & 99 & 99 & 99 & 99 & 99 & 99 & 99 \end{bmatrix}$$

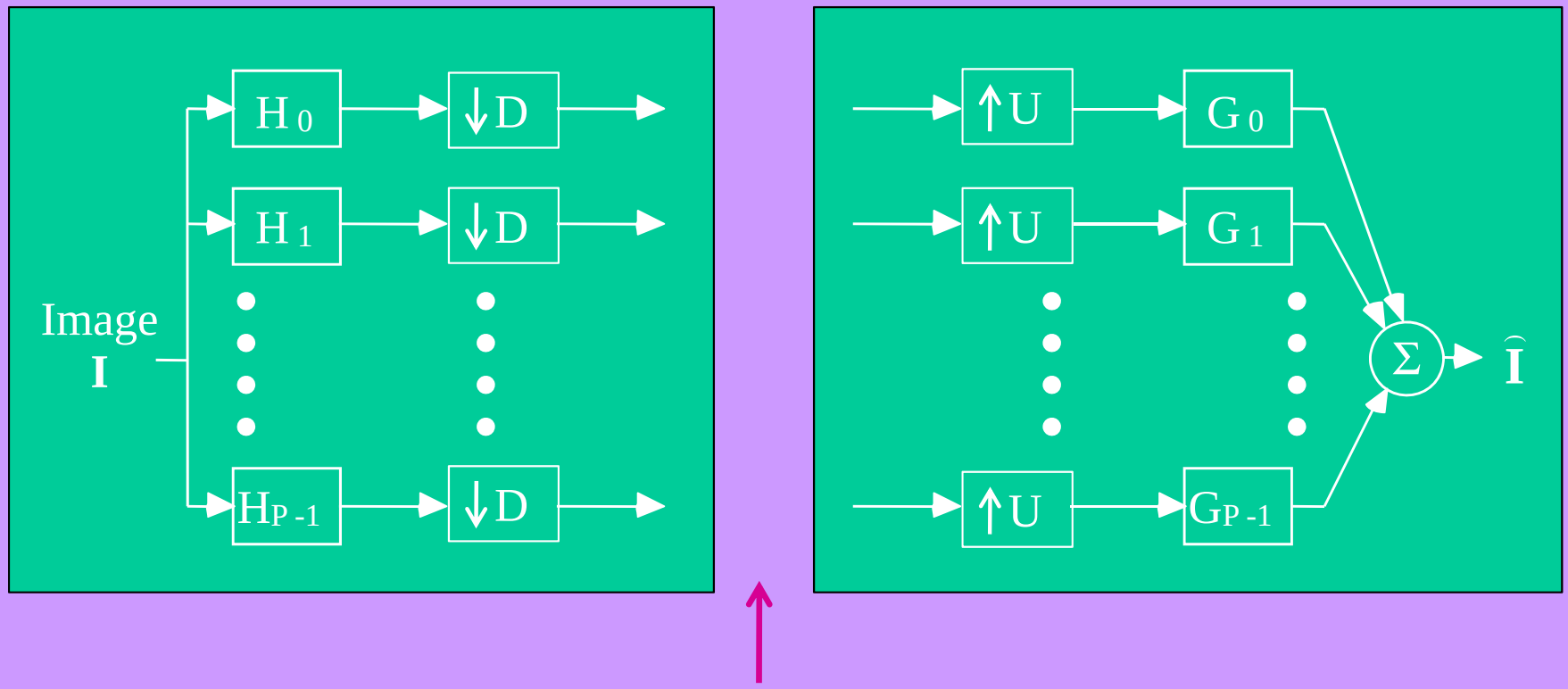
- **Sparser sampling** and more **severe quantization** of the chrominance values is acceptable.
- Color is primarily a **regional attribute** and contains less detail information.

Wavelet Image Compression

Wavelet Image Coding

- **Idea**: Compress images in the wavelet domain.
- Image blocks are not used. Hence: **no blocking artifacts!**
- Forms the basis of the **JPEG2000 standard**.
- Also requires **perfect reconstruction** filter banks.

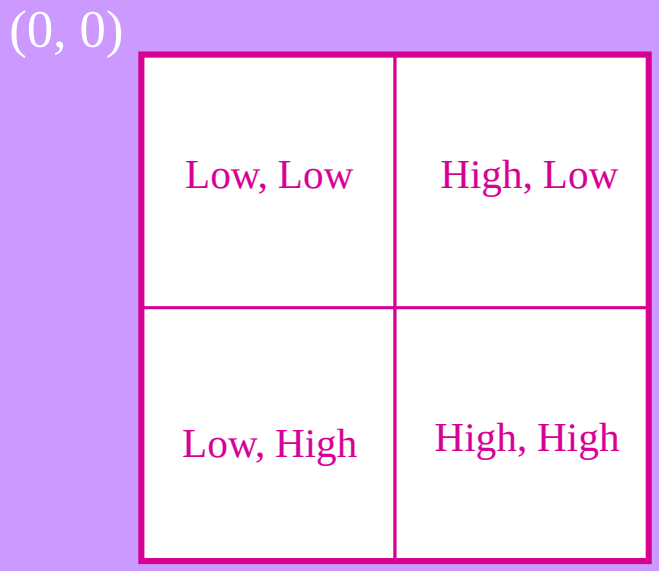
Compression Via DWT



Filters H_p form a **orthogonal or bi-orthogonal basis** with reconstruction filters G_p.

Wavelet Decompositions

- Recall that wavelet filters **separate or decompose** the image into high- and low-frequency bands, called **subbands**. Each frequency band can be coded **separately**.

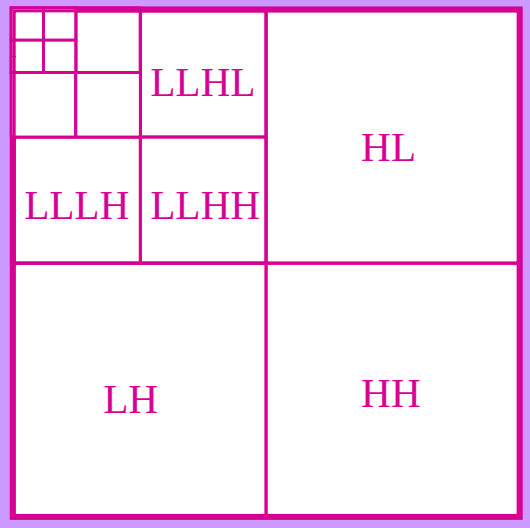


- These are called **quadrature filters**.

Wavelet Decompositions

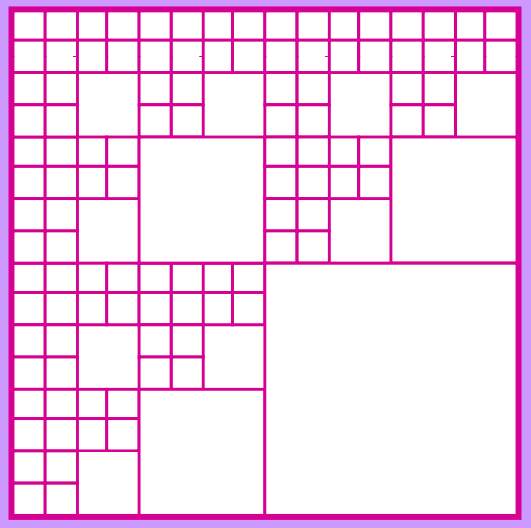
- Typically frequency division is done iteratively on the **lowest-frequency bands**:

(0, 0)



"Pyramid" Wavelet Transform

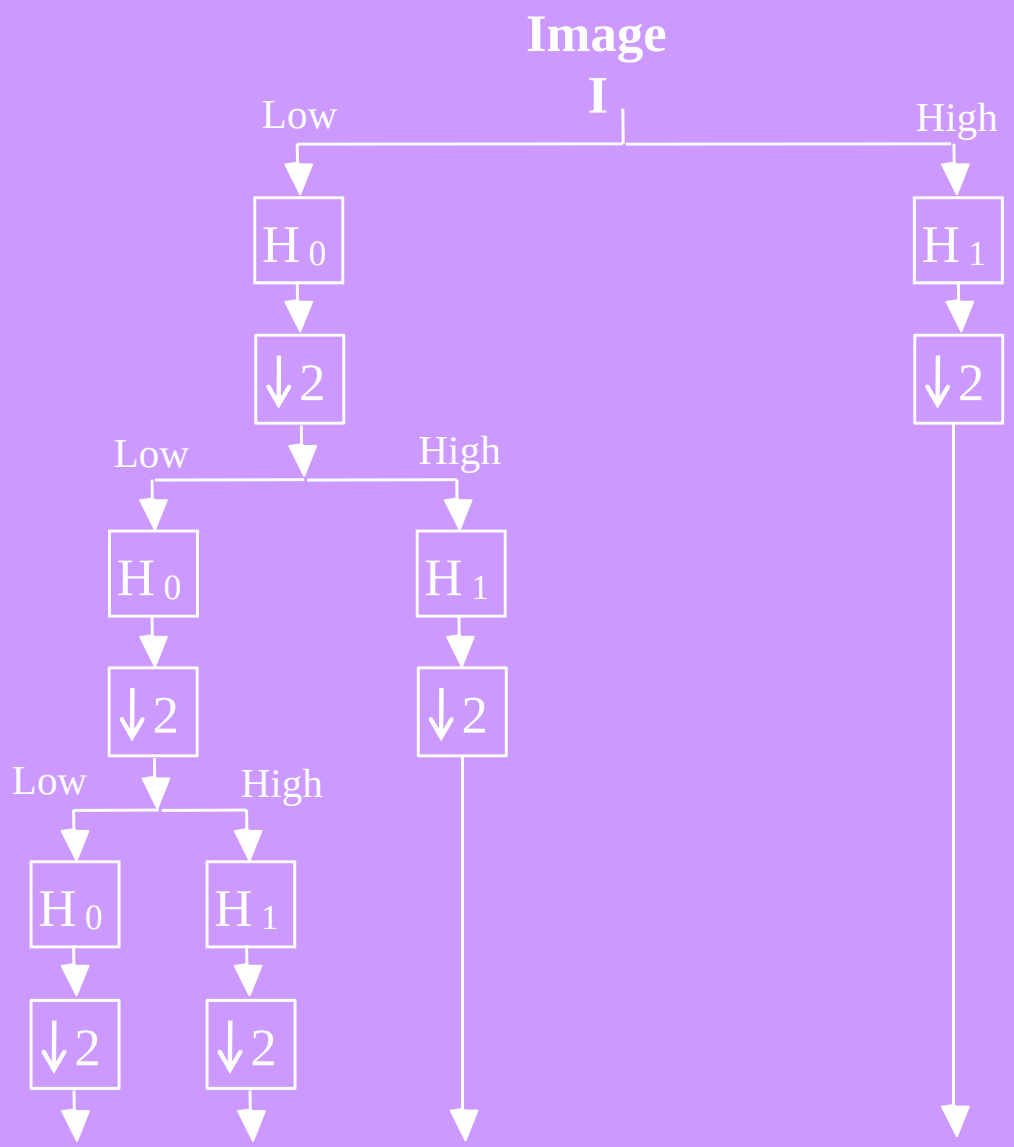
(0, 0)



"Tree-Structured" Wavelet Transform

Wavelet Filter Hierarchy

- **Filter outputs** are **wavelet coefficients.**
- **Subsampling** yields NM **non-redundant** coefficients. The image can be **exactly** reconstructed from them.
- **Idea:** Wavelet coefficients **quantized** (like JPEG): **higher frequency coefficients** quantized **more severely.**
- The **JPEG2000 Standard** is wavelet based.



Bi-Orthogonal Wavelets

- Recall the **perfect reconstruction condition:**

$$\underset{\sim}{H}_0(\omega)\underset{\sim}{G}_0(\omega) + \underset{\sim}{H}_1(\omega)\underset{\sim}{G}_1(\omega) = 2 \quad \text{for all } 0 \le \omega \le \pi \quad \text{(DSFT)}$$

hence also $\tilde{H}_0(u)\tilde{G}_0(u) + \tilde{H}_1(u)\tilde{G}_1(u) = 2$ for all $0 \le u \le N\text{-}1$ (DFT)

and

$$\underset{\sim}{H}_0(\omega+\pi)\underset{\sim}{G}_0(\omega) + \underset{\sim}{H}_1(\omega+\pi)\underset{\sim}{G}_1(\omega) = 0 \quad \text{for all } 0 \le \omega \le \pi \quad \text{(DSFT)}$$

hence also $\tilde{H}_0(u+N/2)\tilde{G}_0(u) + \tilde{H}_1(u+N/2)\tilde{G}_1(u) = 0$ for all $0 \le u \le N\text{-}1$ (DFT)

- The filters H_0, H_1, G_0, G_1 can be either **orthogonal quadrature mirror filters,** or they can be bi-orthogonal

$$\sum_{i=1}^{N-1} H_n(i)\,G_m(i) = \begin{cases} 1; & m = n \\ 0; & m \neq n \end{cases}$$

for all m, n = 0, 1.

Bi-Orthogonal Wavelets

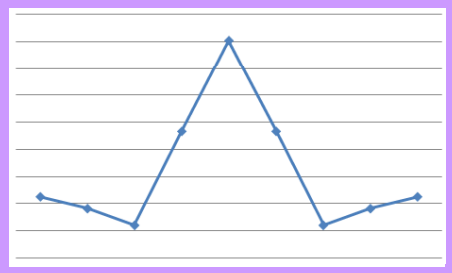
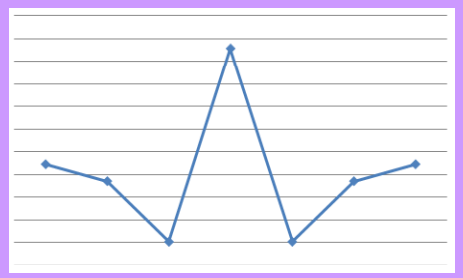
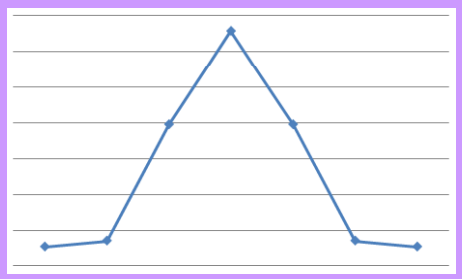
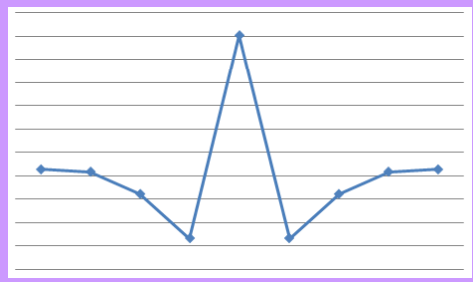
- Bi-orthogonal wavelets **are useful since they can be made even-symmetric.**
- This is important because much **better results** can be obtained in **image compression.**
- Even though they **do not** have as **good energy compaction** as **orthogonal wavelets**.

Bi-Orthogonal Wavelets

- Wavelet based image compression usually operates on **large image blocks** (e.g., 64x64 in JPEG2000)
- It is much more efficient to realize block DWTs using **cyclic convolutions** (shorter).
- As with the DCT, **symmetric extended** signals (without discontinuities) give better results than **periodic extended.**
- **Symmetric wavelets** give **better approximations to symmetric extended** signals.
- There are **standard bi-orthogonal wavelets**.

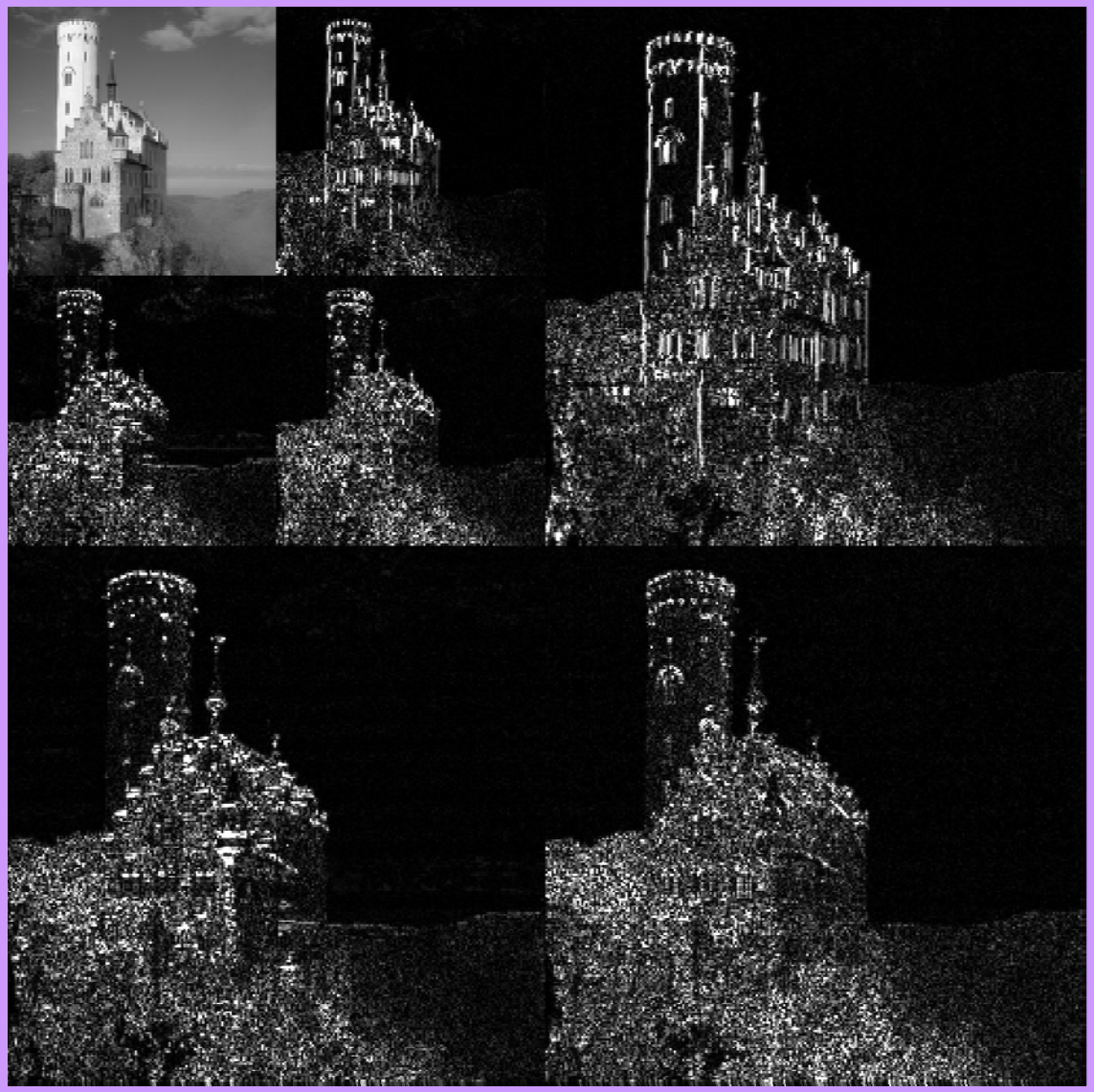
Daubechies 9/7 Bi-Orthogonal Wavelets

Analysis LP	Analysis HP	Synthesis LP	Synthesis HP
$H_0(0) = 0.026749$	$H_0(0) = 0$	$G_0(0) = 0$	$G_1(0) = 0.026749$
$H_0(1) = -0.016864$	$H_1(1) = 0.091272$	$G_0(1) = -0.091272$	$G_1(1) = 0.016864$
$H_0(2) = -0.078223$	$H_1(2) = -0.05754$	$G_0(2) = -0.05754$	$G_1(2) = -0.078223$
$H_0(3) = 0.266864$	$H_1(3) = -0.59127$	$G_0(3) = 0.59127$	$G_1(3) = -0.266864$
$H_0(4) = 0.602949$	$H_1(4) = 1.115087$	$G_0(4) = 1.115087$	$G_1(4) = 0.602949$
$H_0(5) = 0.266864$	$H_1(5) = -0.59127$	$G_0(5) = 0.59127$	$G_1(5) = -0.266864$
$H_0(6) = -0.078223$	$H_1(6) = -0.05754$	$G_0(6) = -0.05754$	$G_1(6) = -0.078223$
$H_0(7) = -0.016864$	$H_1(7) = 0.091272$	$G_0(7) = -0.091272$	$G_1(7) = 0.016864$
$H_0(8) = 0.026749$	$H_1(8) = 0$	$G_0(8) = 0$	$G_1(8) = 0.026749$

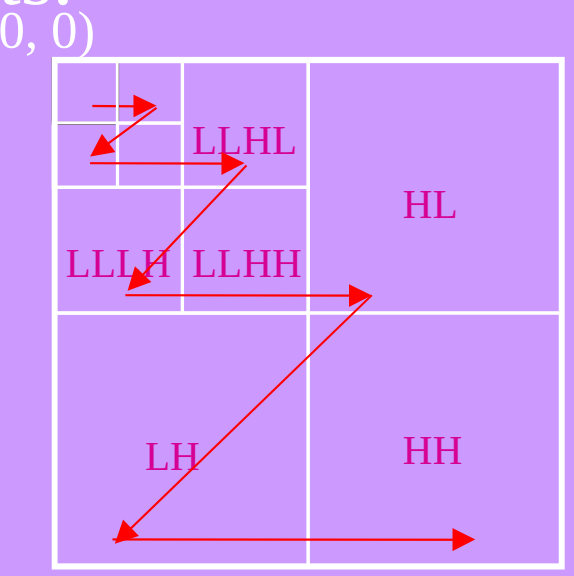
These are used in the **JPEG2000 standard.**

D9/7 DWT of an Image



Zero-Tree Encoding Concept

- There are **many proposed** wavelet-based image compression techniques. Many are **complex.**
- A simple concept is **zero-tree encoding:** Similar to JPEG, **scan from lower to higher frequencies** and **run-length code** the zero coefficients.



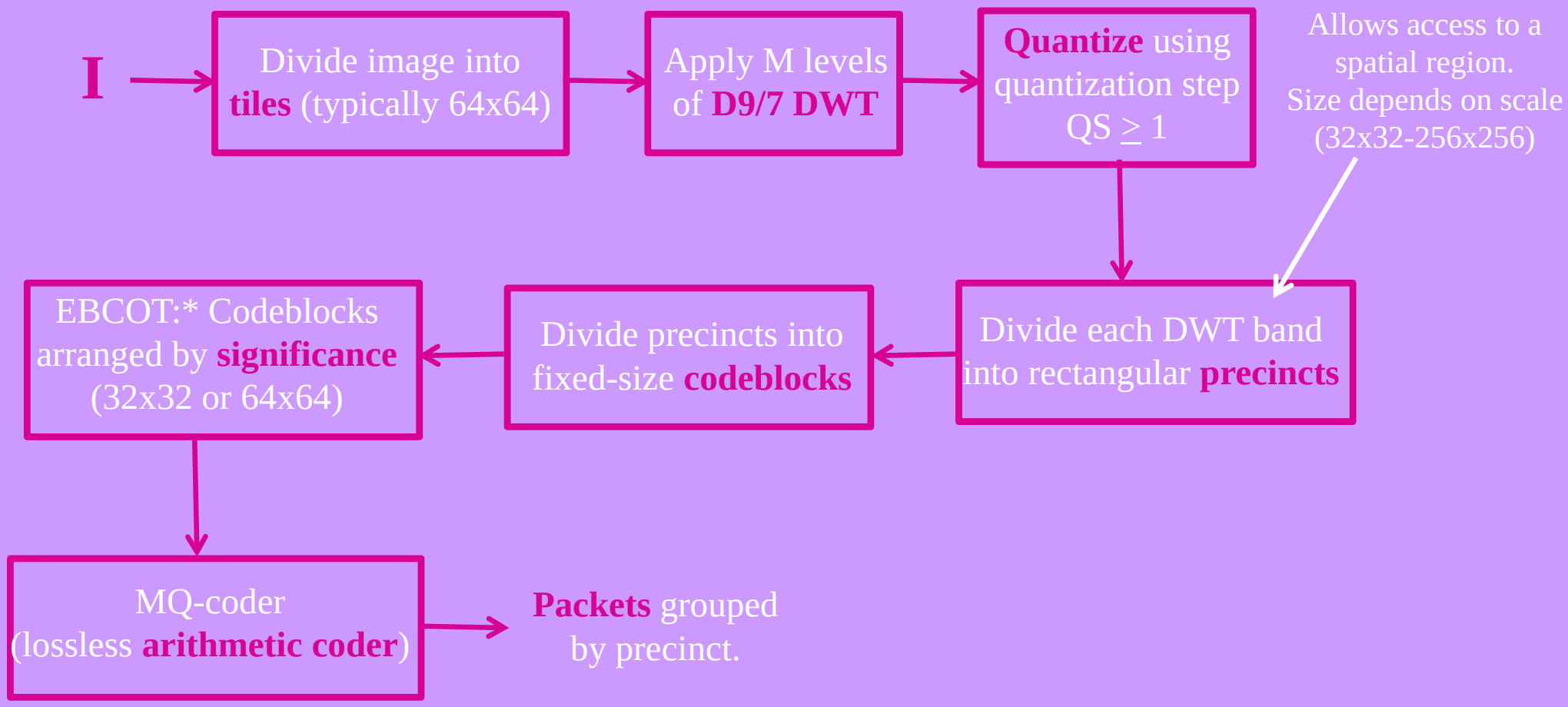
Zig-zag scanning to find zeroed coefficients

JPEG2000

- **Too complex** to go into depth, and **not widely-accepted** enough to spend much time on.

- **Advantages:**
  - Better compression at low bitrates
  - Scalable (can truncate bitstream at any point)
  - No Blocking artifacts
- **Disadvantages:**
  - Complex encoders/decoders
  - Only slightly better compression at high bitrates
  - Ringing artifacts

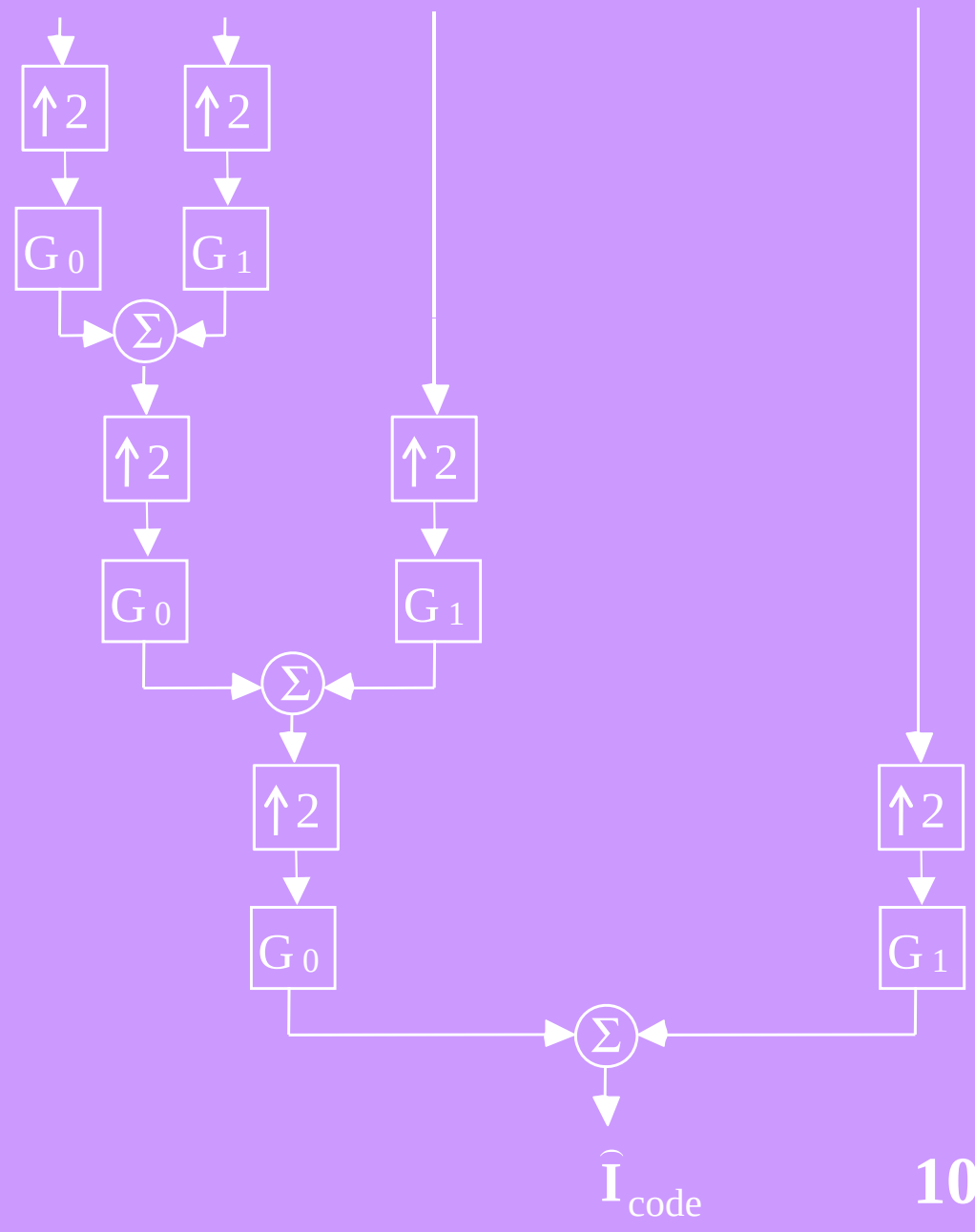
JPEG2000



*Embedded Block Coding with Optimal Truncation

Wavelet Decoding

- **Decoding** requires
  - Decompression of the wavelet coefficients
  - Reconstructing the image from the decoded coefficients:



JPEG2000 Wavelet Compression Example

Original
512 x 512
Barbara

16 : 1

32 : 1

64 : 1

128 : 1

Original
512 x 512
Boats

16 : 1

32 : 1

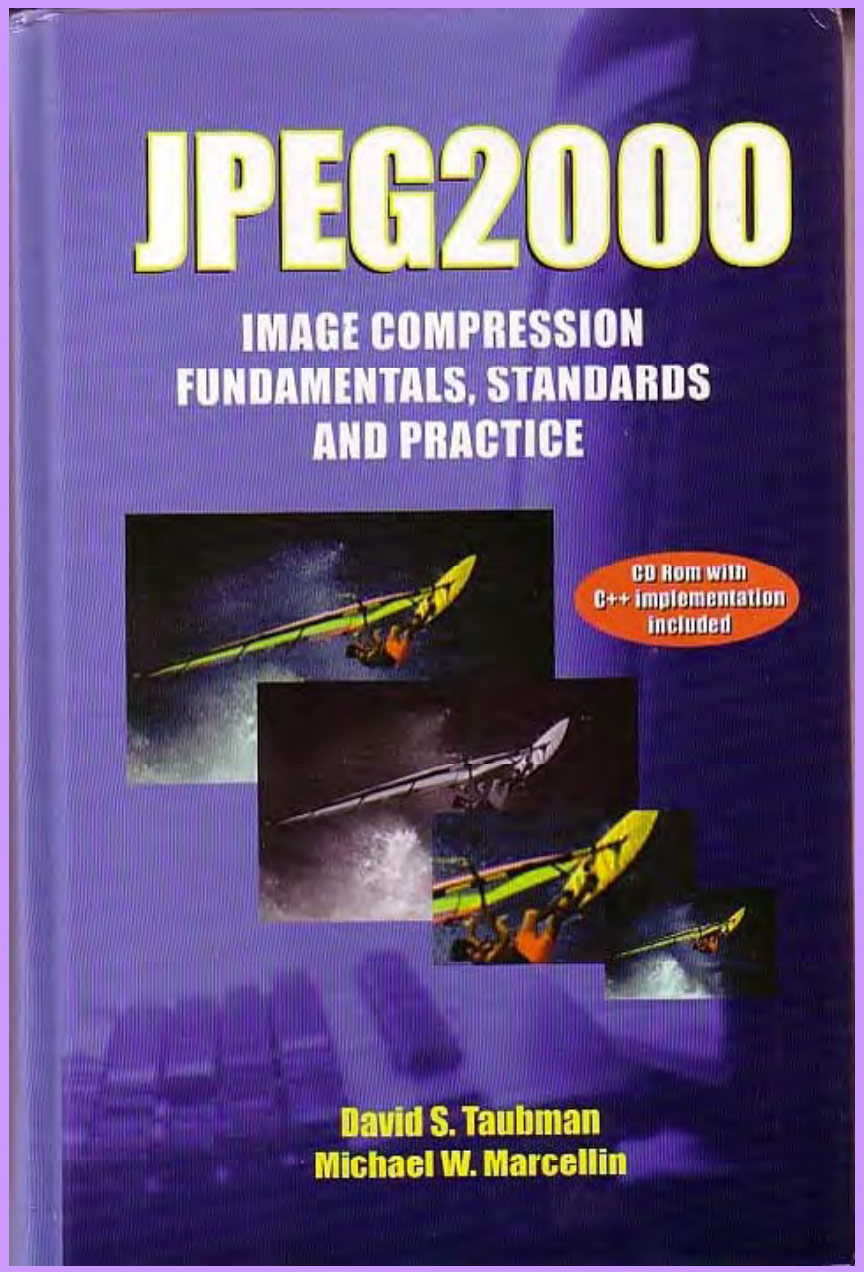
64 : 1

128 : 1

Comments on Wavelet Compression

- This has been a **watered-down** exposition of wavelet coding!
- **Reason:** there has not yet been much adoption of JPEG2000.
- Why? It is **vastly** more complex than "old" JPEG. A consideration in an era of **cheap memory**.
- There are questions of **propriety.**
- It's performance is primarily better in the **high compression** regime.

- If interested….



Comments

- So far we have been **processing images** – now we will try to **understand** them – **image analysis** … onward to **Module 7**.