A1 2014 p. 3.53: (A1)

$$\underset{\sim}{I}_c(\Omega,\Lambda) = \int_{\mathbb{R}^2} I_c(x,y)\, e^{-j(x\Omega + y\Lambda)}\, dx\,dy$$

$$\boxed{\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}}$$

$$I_c(x,y) = \frac{1}{(2\pi)^2}\int_{\mathbb{R}^2} \underset{\sim}{I}_c(\Omega,\Lambda)\, e^{j(x\Omega + y\Lambda)}\, d\Omega\, d\Lambda$$

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p. 3.73: $I_c(x,y) = c \cdot \operatorname{rect}\left(\frac{x}{A}\right)\operatorname{rect}\left(\frac{y}{B}\right) = \begin{cases} c, & |x| \le \frac{A}{2},\ |y| \le \frac{B}{2} \\ 0, & \text{otherwise} \end{cases}$

$$\operatorname{rect}(x) = \begin{cases} 1, & |x| \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

$\boxed{\text{assume } A, B > 0}$

$$\operatorname{rect}\left(\frac{x}{A}\right) = \begin{cases} 1, & \left|\frac{x}{A}\right| \le \frac{1}{2} \\ 0, & \text{other} \end{cases} = \begin{cases} 1, & \frac{|x|}{A} \le \frac{1}{2} \\ 0, & \text{other} \end{cases} = \begin{cases} 1, & |x| \le \frac{A}{2} \\ 0, & \text{other} \end{cases} \checkmark$$

$$\underset{\sim}{I}_c(\Omega,\Lambda) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_c(x,y)\, e^{-j(x\Omega + y\Lambda)}\, dx\,dy$$

$$= c\int_{-B/2}^{B/2}\int_{-A/2}^{A/2} e^{-j\Omega x} e^{-j\Lambda y}\, dx\,dy = c\left[\int_{-A/2}^{A/2} e^{-j\Omega x}\,dx\right]\left[\int_{-B/2}^{B/2} e^{-j\Lambda y}\,dy\right]$$

$$= c\left(\frac{-1}{j\Omega}\right) e^{-j\Omega x}\Big|_{x=-\frac{A}{2}}^{\frac{A}{2}} \cdot \left(-\frac{1}{j\Lambda}\right) e^{-j\Lambda y}\Big|_{y=-B/2}^{B/2}$$

$$= c(-1)\,\frac{e^{-j\frac{\Omega A}{2}} - e^{j\frac{\Omega A}{2}}}{j\Omega}\,(-1)\,\frac{e^{-j\frac{\Lambda B}{2}} - e^{j\frac{\Lambda B}{2}}}{j\Lambda}$$

$$= 4c\,\frac{e^{j\frac{\Omega A}{2}} - e^{-j\frac{\Omega A}{2}}}{j2\Omega}\,\frac{e^{j\Lambda B/2} - e^{-j\Lambda B/2}}{j2\Lambda} = 4c\,\frac{\sin(\Omega A/2)}{\Omega}\,\frac{\sin(\Lambda B/2)}{\Lambda}$$

$$= 4c\,\frac{A}{2}\,\frac{\sin(\Omega A/2)}{\Omega A/2}\,\frac{B}{2}\,\frac{\sin(\Lambda B/2)}{\Lambda B/2} = cAB\,\frac{\sin\left[\pi\left(\frac{\Omega A}{2\pi}\right)\right]}{\pi\left(\frac{\Omega A}{2\pi}\right)}\,\frac{\sin\left[\pi\left(\frac{\Lambda B}{2\pi}\right)\right]}{\pi\left(\frac{\Lambda B}{2\pi}\right)}$$

$$= \underline{\underline{cAB\operatorname{sinc}\left(\frac{\Omega A}{2\pi}\right)\operatorname{sinc}\left(\frac{\Lambda B}{2\pi}\right)}}$$

Al 2014 p. 3.74 $\boxed{\text{assume } a,b>0}$ (A2)

$$\text{rect}(x) = \begin{cases} 1, & |x| \le \frac{1}{2} \\ 0, & \text{other} \end{cases} \qquad \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$\text{rect}\left(\frac{\Omega}{2a}\right) = \begin{cases} 1, & \frac{|\Omega|}{2a} \le \frac{1}{2} \\ 0, & \text{other} \end{cases} = \begin{cases} 1, & |\Omega| \le a \\ 0, & \text{other} \end{cases}$$

$$\underset{\sim}{I}_c(\Omega, \Lambda) = c\,\text{rect}\left(\frac{\Omega}{2a}\right)\text{rect}\left(\frac{\Lambda}{2b}\right)$$

$$I_c(x,y) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underset{\sim}{I}_c(\Omega,\Lambda)\, e^{j(x\Omega + y\Lambda)}\, d\Omega\, d\Lambda = \frac{c}{4\pi^2}\int_{-b}^{b}\int_{-a}^{a} e^{j\Omega x} e^{j\Lambda y}\, d\Omega\, d\Lambda$$

$$= \frac{c}{4\pi^2}\left[\int_{-a}^{a} e^{j\Omega x}\, d\Omega\right]\left[\int_{-b}^{b} e^{j\Lambda y}\, d\Lambda\right] = \frac{c}{4\pi^2}\left[\frac{1}{jx} e^{j\Omega x}\right]_{\Omega=-a}^{a}\left[\frac{1}{jy} e^{j\Lambda y}\right]_{\Lambda=-b}^{b}$$

$$= \frac{c}{4\pi^2}\,\frac{e^{jax} - e^{-jax}}{jx}\,\frac{e^{jby} - e^{-jby}}{jy} = \frac{abc}{\pi^2}\,\frac{e^{jax} - e^{-jax}}{2j\,ax}\,\frac{e^{jby} - e^{-jby}}{2j\,by}$$

$$= \frac{abc}{\pi^2}\,\frac{e^{j\pi\frac{ax}{\pi}} - e^{-j\pi\frac{ax}{\pi}}}{2j\,\pi\frac{ax}{\pi}}\,\frac{e^{j\pi\frac{by}{\pi}} - e^{-j\pi\frac{by}{\pi}}}{2j\,\pi\frac{by}{\pi}}$$

$$= \frac{abc}{\pi^2}\,\frac{\sin\left[\pi\left(\frac{ax}{\pi}\right)\right]}{\pi\frac{ax}{\pi}}\,\frac{\sin\left[\pi\left(\frac{by}{\pi}\right)\right]}{\pi\frac{by}{\pi}} = \underline{\underline{\frac{abc}{\pi^2}\,\text{sinc}\left(\frac{ax}{\pi}\right)\text{sinc}\left(\frac{by}{\pi}\right)}}$$

So $\frac{abc}{\pi^2}\,\text{sinc}\left(\frac{ax}{\pi}\right)\text{sinc}\left(\frac{by}{\pi}\right) \overset{\mathcal{F}}{\longleftrightarrow} c\cdot\text{rect}\left(\frac{\Omega}{2a}\right)\text{rect}\left(\frac{\Lambda}{2b}\right)$

$$\frac{c}{\pi^2}\,\text{sinc}\left(\frac{ax}{\pi}\right)\text{sinc}\left(\frac{by}{\pi}\right) \overset{\mathcal{F}}{\longleftrightarrow} \frac{c}{ab}\,\text{rect}\left(\frac{\Omega}{2a}\right)\text{rect}\left(\frac{\Lambda}{2b}\right)$$

$\rightarrow$ Let $A = 2a$, $B = 2b$ $\mapsto$ $a = \frac{A}{2}$, $b = \frac{B}{2}$

$$\frac{c}{\pi^2}\,\text{sinc}\left(\frac{Ax}{2\pi}\right)\text{sinc}\left(\frac{By}{2\pi}\right) \overset{\mathcal{F}}{\longleftrightarrow} \frac{4c}{AB}\,\text{rect}\left(\frac{\Omega}{A}\right)\text{rect}\left(\frac{\Lambda}{B}\right)$$

$\rightarrow$ write a,b for A,B

$$\star\ \underline{\underline{\frac{c}{(2\pi)^2}\,\text{sinc}\left(\frac{ax}{2\pi}\right)\text{sinc}\left(\frac{by}{2\pi}\right) \overset{\mathcal{F}}{\longleftrightarrow} \frac{c}{ab}\,\text{rect}\left(\frac{\Omega}{a}\right)\text{rect}\left(\frac{\Lambda}{b}\right)}}$$

p. 3.75 $I_c(x,y) = \exp\left[-(x^2+y^2)/\sigma^2\right]$

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$$\mathcal{I}_c(\Omega,\Lambda) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_c(x,y)\, e^{-j(x\Omega + y\Lambda)}\, dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2/\sigma^2} e^{-y^2/\sigma^2} e^{-j\Omega x} e^{-j\Lambda y}\, dx\,dy$$

$$= \left[\int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{\sigma^2} - j\Omega x\right) dx\right]\left[\int_{-\infty}^{\infty} \exp\left(-\frac{y^2}{\sigma^2} - j\Lambda y\right) dy\right]$$

$$\int_{-\infty}^{\infty} \exp(-ax^2 - bx - c)\, dx = \left(\frac{\pi}{a}\right)^{1/2} \exp\left(\frac{b^2-4ac}{4a}\right)$$

$$a = \frac{1}{\sigma^2},\ b = j\Omega,\ c = 0$$

$$= (\pi\sigma^2)^{1/2} \exp\left[\frac{-\Omega^2}{4/\sigma^2}\right] (\pi\sigma^2)^{1/2} \exp\left[\frac{-\Lambda^2}{4/\sigma^2}\right] = \pi\sigma^2 \exp\left[\frac{-(\Omega^2+\Lambda^2)}{4}\sigma^2\right]$$

$$= \pi\sigma^2 \exp\left[-\frac{\sigma^2}{4}(\Omega^2+\Lambda^2)\right]$$

p. 4.112

$$\tilde{I}_c(\Omega,\Lambda) = \int_{\mathbb{R}^2} I_c(x,y)\, e^{-j2\pi(\Omega x + \Lambda y)}\, dx\,dy$$ (J1)

$$I_c(x,y) = \int_{\mathbb{R}^2} \tilde{I}_c(\Omega,\Lambda)\, e^{j2\pi(\Omega x + \Lambda y)}\, d\Omega\, d\Lambda$$

$$\boxed{\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}}$$

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p. 4.131 $$\operatorname{rect}(x) = \begin{cases} 1, & |x| \le \frac{1}{2} \\ 0, & \text{other} \end{cases}$$

$\boxed{\text{assume } A, B > 0}$

$$\operatorname{rect}\left(\frac{x}{A}\right) = \begin{cases} 1, & \frac{|x|}{A} \le \frac{1}{2} \\ 0, & \text{other} \end{cases} = \begin{cases} 1, & |x| \le \frac{A}{2} \\ 0, & \text{other} \end{cases} \checkmark$$

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p. 4.131 $$I_c(x,y) = c\,\operatorname{rect}\left(\frac{x}{A}\right)\operatorname{rect}\left(\frac{y}{B}\right) = \begin{cases} c, & |x| \le \frac{A}{2},\ |y| \le \frac{B}{2} \\ 0, & \text{other} \end{cases}$$

$$\tilde{I}_c(\Omega,\Lambda) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_c(x,y)\, e^{-j2\pi(\Omega x + \Lambda y)}\, dx\,dy$$

$$= c\int_{-B/2}^{B/2}\int_{-A/2}^{A/2} e^{-j2\pi\Omega x} e^{-j2\pi\Lambda y}\, dx\,dy = c\left[\int_{-A/2}^{A/2} e^{-j2\pi\Omega x}\, dx\right]\left[\int_{-B/2}^{B/2} e^{-j2\pi\Lambda y}\, dy\right]$$

$$= c\, \frac{-1}{j2\pi\Omega} e^{-j2\pi\Omega x}\Big|_{x=-\frac{A}{2}}^{\frac{A}{2}} \cdot \frac{-1}{j2\pi\Lambda} e^{-j2\pi\Lambda y}\Big|_{y=-B/2}^{B/2}$$

$$= c\, \frac{e^{-j2\pi\frac{\Omega A}{2}} - e^{j2\pi\frac{\Omega A}{2}}}{-j2\pi\Omega} \cdot \frac{e^{-j2\pi\frac{\Lambda B}{2}} - e^{j2\pi\frac{\Lambda B}{2}}}{-j2\pi\Lambda}$$

$$= c\, \frac{e^{j\pi\Omega A} - e^{-j\pi\Omega A}}{2j\,\pi\Omega} \cdot \frac{e^{j\pi\Lambda B} - e^{-j\pi\Lambda B}}{2j\,\pi\Lambda} = c\, \frac{\sin(\pi\Omega A)}{\pi\Omega A} A\, \frac{\sin(\pi\Lambda B)}{\pi\Lambda} B$$

$$= cAB\operatorname{sinc}(A\Omega)\operatorname{sinc}(B\Lambda) \checkmark$$

J2

$$\mathrm{rect}(x) = \begin{cases} 1, & |x| \le \frac{1}{2} \\ 0, & \text{other} \end{cases} \qquad \mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$\mathrm{rect}\left(\frac{x}{a}\right) = \begin{cases} 1, & |x| \le \frac{a}{2} \\ 0, & \text{other} \end{cases}$$

p. 4.132: $I_c(x,y) = c\,\mathrm{sinc}(ax)\,\mathrm{sinc}(by)$

$$\tilde{I}_c(\Omega,\Lambda) = \frac{c}{ab}\,\mathrm{rect}\left(\frac{\Omega}{a}\right)\mathrm{rect}\left(\frac{\Lambda}{b}\right)$$

$$I_c(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{I}_c(\Omega,\Lambda)\, e^{j2\pi(\Omega x + \Lambda y)}\, d\Omega\, d\Lambda$$

$$= \frac{c}{ab}\int_{-b/2}^{b/2}\int_{-a/2}^{a/2} e^{j2\pi\Omega x} e^{j2\pi\Lambda y}\, d\Omega\, d\Lambda = \frac{c}{ab}\left[\int_{-a/2}^{a/2} e^{j2\pi\Omega x}\, d\Omega\right]\left[\int_{-b/2}^{b/2} e^{j2\pi\Lambda y}\, d\Lambda\right]$$

$$= \frac{c}{ab}\,\frac{1}{j2\pi x}\, e^{j2\pi\Omega x}\Big|_{\Omega=-a/2}^{a/2} \cdot \frac{1}{j2\pi y}\, e^{j2\pi\Lambda y}\Big|_{\Lambda=-b/2}^{b/2}$$

$$= \frac{c}{ab}\,\frac{e^{j2\pi\frac{ax}{2}} - e^{-j2\pi\frac{ax}{2}}}{j2\pi x}\cdot\frac{e^{j2\pi\frac{by}{2}} - e^{-j2\pi\frac{by}{2}}}{j2\pi y} = \frac{c}{ab}\,\frac{e^{j\pi ax} - e^{-j\pi ax}}{2j\ \pi x}\,\frac{e^{j\pi by} - e^{-j\pi by}}{2j\ \pi y}$$

$$= \frac{c}{ab}\,\frac{\sin(\pi ax)}{\pi ax}\,a\,\frac{\sin(\pi by)}{\pi by}\,b = c\,\frac{\sin(\pi ax)}{\pi ax}\,\frac{\sin(\pi by)}{\pi by}$$

$$= c\,\mathrm{sinc}(ax)\,\mathrm{sinc}(by) \checkmark$$

P. 4.133 $I_c(x,y) = \exp\left[\frac{-(x^2+y^2)}{\sigma^2}\right]$ (J3)

$$\tilde{I}_c(\Omega,\Lambda) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I_c(x,y)\, e^{-j2\pi(\Omega x+\Lambda y)}\, dx\, dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2/\sigma^2} e^{-y^2/\sigma^2} e^{-j2\pi\Omega x} e^{-j2\pi\Lambda y}\, dx\, dy$$

$$= \left[\int_{-\infty}^{\infty} e^{-x^2/\sigma^2} e^{-j2\pi\Omega x}\, dx\right]\left[\int_{-\infty}^{\infty} e^{-y^2/\sigma^2} e^{-j2\pi\Lambda y}\, dy\right]$$

$$= \left[\int_{-\infty}^{\infty} \exp\left(-\frac{1}{\sigma^2}x^2 - j2\pi\Omega x\right) dx\right]\left[\int_{-\infty} \exp\left(-\frac{1}{\sigma^2}y^2 - j2\pi\Lambda y\right) dy\right]$$

$$\int_{-\infty}^{\infty} \exp(-ax^2 - bx - c)\, dx = \left(\frac{\pi}{a}\right)^{1/2} \exp\left(\frac{b^2-4ac}{4a}\right)$$

$$a = \frac{1}{\sigma^2} \quad b = j2\pi\Omega \quad c = 0$$

$$= (\pi\sigma^2)^{1/2} \exp\left[\frac{-4\pi^2\Omega^2}{4/\sigma^2}\right] (\pi\sigma^2)^{1/2} \exp\left[\frac{-4\pi^2\Lambda^2}{4/\sigma^2}\right]$$

$$= (\pi^2\sigma^4)^{1/2} \exp\left[-\pi^2\Omega^2\sigma^2 - \pi^2\Lambda^2\sigma^2\right]$$

$$= \pi\sigma^2 \exp\left[-\pi^2\sigma^2(\Omega^2+\Lambda^2)\right] \checkmark$$