Steerable Pyramid

Chuong Nguyen

November 19, 2011

Contents

0.1 Introduction

The steerable pyramid is a multi-resolution signal processing scheme. The input signal is decomposed into components whose frequency supports lie in a finite portion of the original's. This processing scheme enables us to analyze the signal in a coarse to fine manner.

One can extract important information of signal such as texture orientations and edge from the oriented filters [**?**].

The steerable pyramid was proposed by Simoncelli *et al.* [1, 2].

Steerability refers to the ability to synthesize filters at an direction as a linear combinations of a small set of basis filters [?].

Let $S = \sum_{k=0}^{K-1} \cos^{2\ell}(\phi - \frac{\pi k}{K})$,

$$\begin{aligned} S &= \frac{1}{2^{2\ell}} \sum_{k=0}^{K-1} \left[e^{i(\phi - \frac{\pi k}{K})} + e^{-i(\phi - \frac{\pi k}{K})} \right]^{2l} \\ &= \frac{1}{2^{2l}} \sum_{k=0}^{K-1} \sum_{m=0}^{2l} \binom{2\ell}{m} e^{i(\phi - \frac{\pi k}{K})m} \times e^{-i(\phi - \frac{\pi k}{K})(2\ell - m)} \\ &= \frac{1}{2^{2l}} \sum_{k=0}^{K-1} \sum_{m=0}^{2l} \binom{2\ell}{m} e^{i2\phi(m-\ell)} \times e^{i(\ell - m)\frac{2\pi k}{K}} \\ &= \frac{1}{2^{2\ell}} \sum_{m=0}^{2\ell} \binom{2\ell}{m} e^{i2\phi(m-\ell)} \times \sum_{k=0}^{K-1} e^{i(\ell - m)\frac{2\pi k}{K}}. \end{aligned}$$

For $\ell = m$, S can be reduced to

$$S = \frac{K}{2^{2\ell}} \cdot \binom{2\ell}{\ell} = \frac{K}{2^{2\ell}} \left[\frac{(2\ell)!}{\ell!(\ell!)} \right].$$

For $l \neq m$, let the left sum in S be $A = \sum_{k=0}^{K-1} e^{i(l-m)\frac{2\pi k}{K}}$. We can see that A is a geometric series

$$A = \frac{1 - e^{i(\ell - m)2\pi}}{1 - e^{\frac{i(\ell-m)2\pi}{K}}}.$$

We can observe that $A = 0$ if $\ell - m \neq nK, n \in \mathcal{Z}$. If we choose $K > \ell$, $A = 0, \forall \ell, m \in \mathcal{Z}$.

Given $K > \ell$, the sum S is independent of ϕ, and is defined as

$$S = \frac{K}{2^{2\ell}} \left[\frac{(2\ell)!}{\ell!(\ell!)} \right].$$

Castleman et. al. [3] showed a simple way of designing filters for the steerable pyramid. They used the 1-D cosine curve

$$LP(a, b, f) = \begin{cases} 1 & f \leq a \\ \sqrt{\frac{1}{2}\left[1 + \cos(\pi \frac{f-a}{b-a})\right]} & a < f < b \\ 0 & f \geq b \end{cases}$$

to construct an interpolation table to design the lowpass and the highpass filter.

Bibliography

[1] E. P. Simoncelli, W. T. Freeman, E. H. Adelson, and D. J. Heeger, "Shiftable multi-scale transform," *IEEE Trans. Info. Theory*, vol. 38, no. 2, pp. 587–607, March. 1992.

[2] E. P. Simoncelli and W. T. Freeman, "The steerable pyramid: a flexible architecture for multi-scale derivative computation," in *Proc. IEEE Int'l. Conf. Image Proc.*, Washington, DC., Oct. 23-26, 1995, pp. 444–447.

[3] M. Schulze K. R. Castleman and Q. Wu, "Simplified design of steerable pyramid filters," in *Proc. Int'l. Symp. Circuit and Syst.*, Montery, CA, May. 31 - Jun. 3, 1998, pp. 329–332.