Modulation Domain Image Processing

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Research Goals

1. Develop a perfect reconstruction AM-FM image transform.
   - Develop a perfect reconstruction (PR) filterbank.
   - Develop a robust FM reconstruction algorithm.
2. Design image processing filters in the modulation domain.
   - AM-based image processing.
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Original Contributions

1. A new modulation domain image filtering framework with two classes of perceptually motivated image filters.
2. A true multi-orientation multi-scale PR filterbank.
3. An artifact-free AM-FM demodulation algorithm.
4. A perfect reconstruction FM algorithm.
5. Extensions of the xAMFM for image decomposition problems.

Organization

1. Part I: Theory
   - The AM-FM Image Model
   - The Multi-scale Multi-orientation PR Filterbank
   - The PR AM-FM Transform
2. Part II: Modulation Domain Image Filtering
   - AM-based Image Filtering
   - FM-based Image Filtering
3. Part III: Extensions and Conclusions

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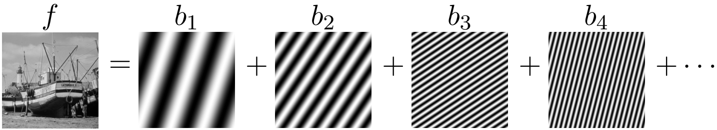
AM-FM Image Model



The AM-FM image model represents an image $f(\mathbf{x})$ as

$$f(\mathbf{x}) = \sum_{k=1}^{K} f_k(\mathbf{x}) = \sum_{k=1}^{K} a_k(\mathbf{x}) \cos[\varphi_k(\mathbf{x})].$$

- $a_k(\mathbf{x})$: amplitude modulation (AM) $\rightarrow$ **local contrast**.
- $\nabla\varphi_k(\mathbf{x})$: frequency modulation (FM) $\rightarrow$ **local texture orientation and spacing**.

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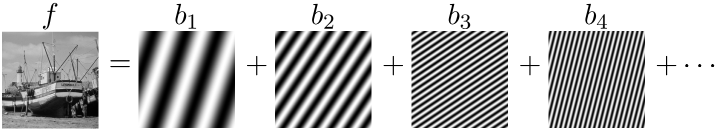

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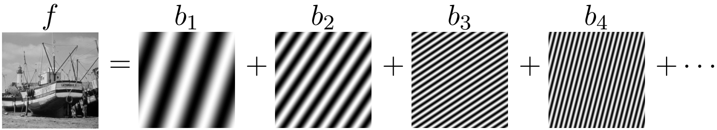

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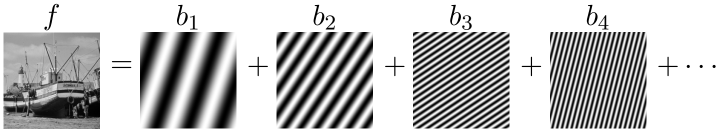

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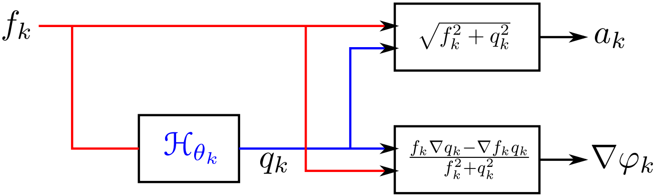
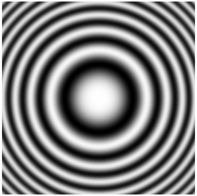
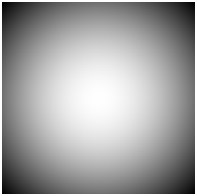
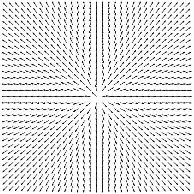
AM-FM Image Model Computation



- $q_k = \mathcal{H}_{\theta_k}[f_k]$.
- a_k and $\nabla \varphi_k$ are computed *exactly*.

- Successfully used in many image processing applications such as image segmentation, content-based retrieval, fingerprint analysis, and target tracking.
- **Problems:** Most AM-FM applications are limited to analysis-only.
  1. Approximation errors of demodulation algorithms.
  2. Lack of perfect reconstruction algorithms.
- **Solution:**
  1. A perfect reconstruction multi-component AM-FM transform.
  2. A robust reconstruction algorithm that produces high quality perceptually motivated filtering results.

The Current State of AM-FM image processing

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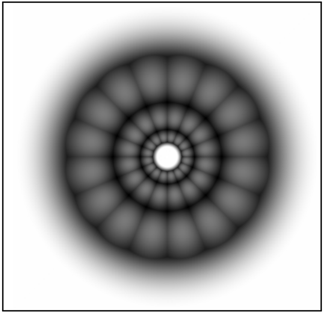
The Steerable Pyramid



- Early 1990s by Freeman, Adelson, and Simoncelli.
- Multi-scale multi-orientation signal analysis.
- Used in many image processing applications.

The Modified Steerable Pyramid: Cont.

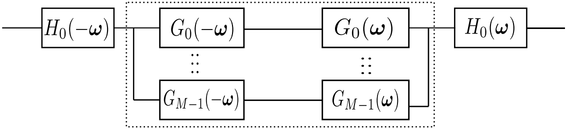
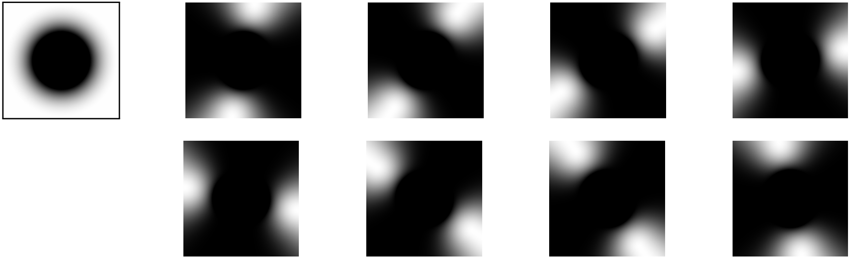


Figure: Hi-pass ring decomposition.



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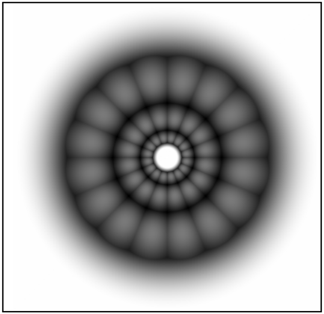


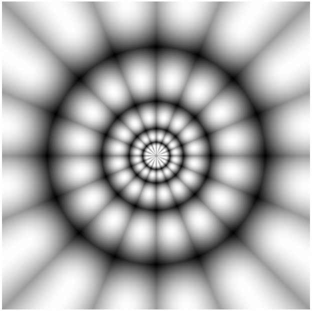


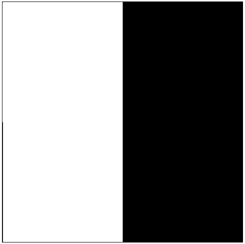
Figure: Original v.s. Modified steerable pyramid

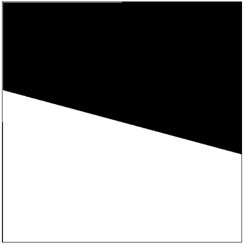
$$\widehat{q}_{k\mathbf{e}}(\boldsymbol{\omega}) = -j\mathrm{sgn}(\boldsymbol{\omega}^T\mathbf{e})\widehat{f}_k(\boldsymbol{\omega}).$$

Problem: locations that have frequency support orthogonal to $\mathbf{e}$

- rippling artifacts in the computed AM.
- distortion in the computed FM.

Solution: rotate the pHT's direction of action such that $\mathbf{e}$ is orthogonal to the steerable pyramid filter center frequency.





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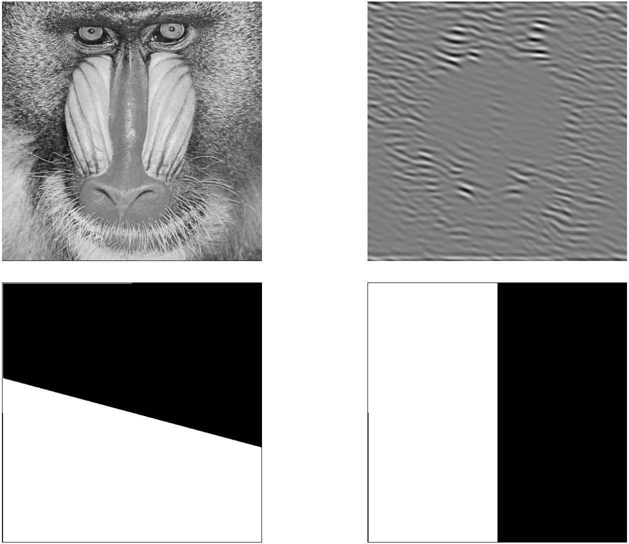

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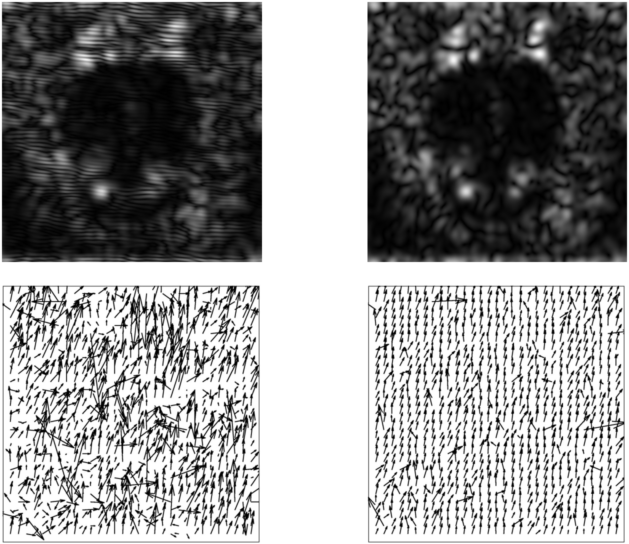

AM-FM Reconstruction Algorithm

Let $f_k(m,n) = a_k(m,n)\cos[\varphi_k(m,n)]$.
Let $\nabla\varphi_k(m,n) = [U(m,n)\ V(m,n)]^T$ be the FM of $f_k(m,n)$.
Sivley and Havlicek proposed a spline-based FM reconstruction.

- Perfect FM reconstruction.
- Cubic tensor product spline.
- Smooth phase function $\varphi_k(m,n)$.

Problems:

1. Reconstructed phase requires a priori knowledge of four pixels.
2. The phase scaling factor problem.
3. Unstable to small changes of $U(m,n)$ and $V(m,n)$.
4. Changes in $U(m,n)$ and $V(m,n)$ are not effectively reflected in the reconstructed phase.

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Let $f_k(m,n) = a_k(m,n)\cos[\varphi_k(m,n)]$.
Let $\psi(m,n)$ be the reconstructed phase function.
Solved using a least-squares approach

$$\mathcal{E}[\psi(m,n)] = ||\psi_m(m,n) - V(m,n)||^2 + ||\psi_n(m,n) - U(m,n)||^2.$$

Problem: $\cos[\psi(m,n)] \neq \cos[\varphi_k(m,n)]$.
Solution:

- $\varphi_k(m,n) = \psi(m,n) + \tau(m,n)$.
- Recompute $\nabla\varphi_k(m,n) = [U(m,n)\ V(m,n)]^T$.

Advantages:

- Reconstructed phase requires a priori knowledge of one pixel.
- Robust to changes in $U(m,n)$ and $V(m,n)$.

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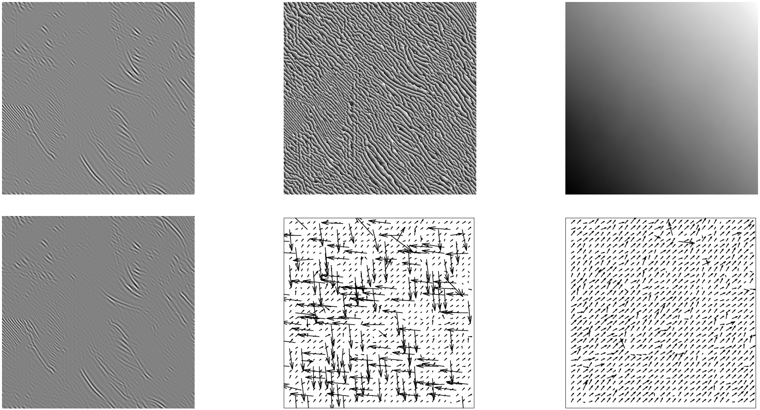

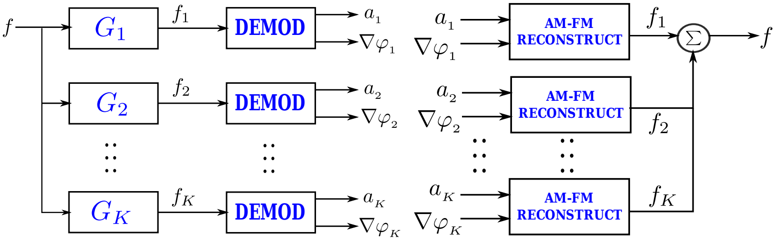

Figure: The xAMFM

Let f be the original image. Let g be the reconstructed image.

$$\begin{aligned}\mathsf{MSE}(f,g) &= \frac{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}[f(m,n)-g(m,n)]^2}{MN},\\ \mathsf{PSNR}(f,g) &= 10\log_{10}\left(\frac{\mathsf{max}[f]}{\mathsf{MSE}(f,g)}\right).\end{aligned}$$

Table: Reconstruction error of the xAMFM.

	Dimension	PSNR (dB)	MSE
Lena	512x512	80.903801	4.874747×10^{-04}
Barbara	512x512	74.902187	1.957273×10^{-03}
Boat	512x512	78.522112	9.138380×10^{-04}
EinSlack	375x500	78.706215	8.085571×10^{-04}
Fingerprint	512x512	74.680474	2.213262×10^{-03}
Flintstones	512x512	73.169935	3.133912×10^{-03}
House	256x256	84.772617	1.903418×10^{-04}
kodim01	512x768	74.077160	2.543105×10^{-03}
kodim05	512x768	73.993926	2.592315×10^{-03}
kodim08	512x768	70.226141	6.172573×10^{-03}
kodim17	768x512	78.422051	9.351371×10^{-04}
kodim22	512x768	77.991403	1.032619×10^{-03}
kodim23	512x768	79.323793	7.598032×10^{-04}

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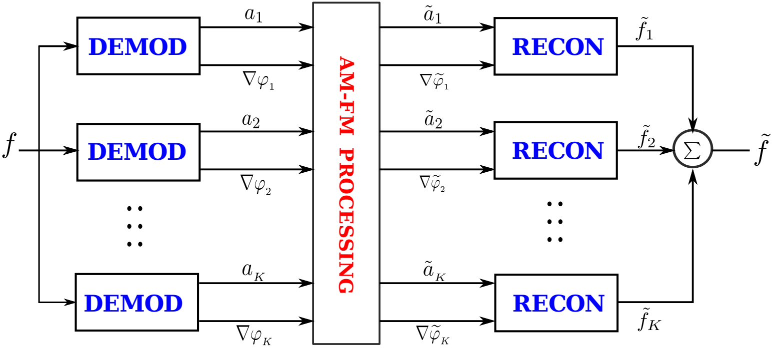

Figure: AM-FM Filtering

AM-based Image Filtering

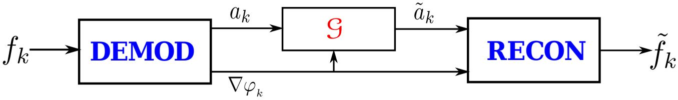

Figure: AM-based Image Filtering

- AM is filtered by filter $\mathcal{G}$.
- FM is unchanged.

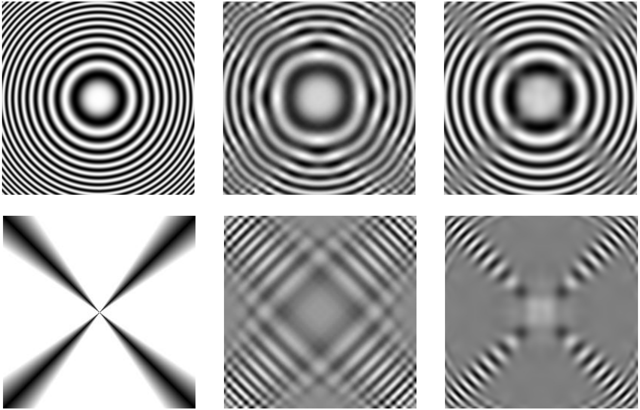

Figure: AM-based selective orientation attenuation.

AM-based Image Filtering: Cont.

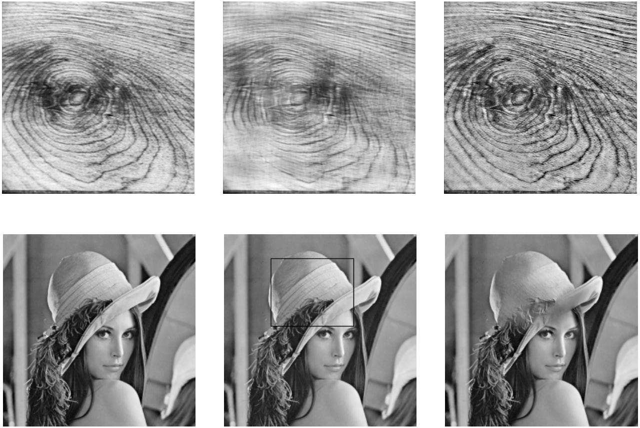

Figure: AM-based image enhancements.

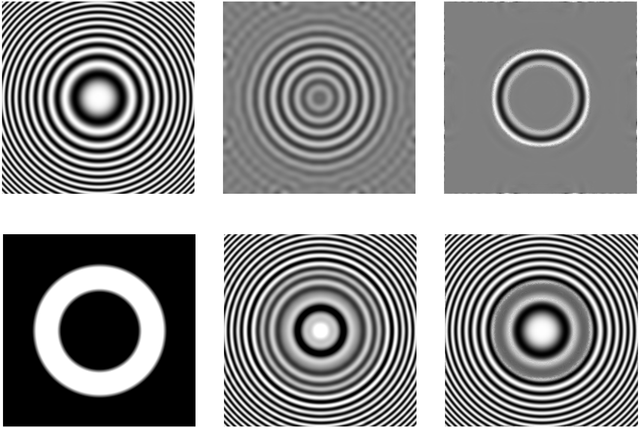

Figure: AM-based bandpass filter.

AM-based Image Fusion

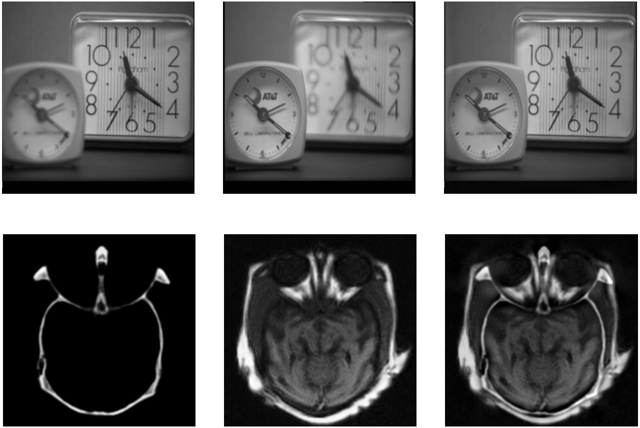

Figure: AM-FM image fusion.

Image Fusion Examples

MI	Wavelet	LP	AM-based
clock	0.577370	**0.630707**	0.610730
tiffany	0.669841	0.64423	**0.671626**
lena	**0.747307**	0.72831	0.704051
medical	**0.413435**	0.365575	0.305687
navigation	**0.242832**	0.25451	0.239581

OEPL	Wavelet	LP	AM-based
clock	0.731238	**0.754339**	0.742081
tiffany	0.735087	**0.739462**	0.738636
lena	**0.741060**	0.739007	0.728825
medical	0.692751	**0.789651**	0.716454
navigation	0.613523	**0.697676**	0.640881

SSIM-based	Wavelet	LP	AM-based
clock	0.490198	**0.509316**	0.502113
tiffany	0.531843	**0.535119**	0.534410
lena	0.548370	**0.549710**	0.544959
medical	**0.301499**	0.279124	0.261661
navigation	0.198293	0.295320	**0.417149**

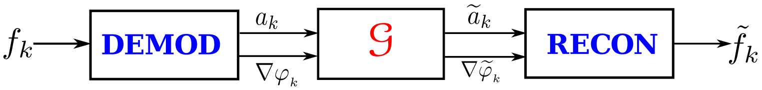

Figure: FM-based Image Filtering

- Both AM and FM functions are filtered.

Let $f_k(\mathbf{n}) = a_k(\mathbf{n})\cos[\varphi_k(\mathbf{n})]$.

- Recall that PR requires $\varphi_k(\mathbf{n}) = \psi_k(\mathbf{n}) + \rho_k(\mathbf{n})$.
- The congruency term $\rho_k(\mathbf{n})$ is the source of artifacts for FM-based filtering.

Solution: move the congruency term $\rho_k(\mathbf{n})$.

- *Generalized* AM functions $A_{1k}(\mathbf{n})$ and $A_{2k}(\mathbf{n})$.
- *Generalized* FM function $\psi_k(\mathbf{n})$.

$$\begin{aligned} f_k(\mathbf{n}) &= a_k(\mathbf{n})\cos[\varphi_k(\mathbf{n})] = a_k(\mathbf{n})\cos[\psi_k(\mathbf{n}) + \rho_k(\mathbf{n})] \\ &= a_k(\mathbf{n})\cos[\rho_k(\mathbf{n})]\cos[\psi_k(\mathbf{n})] - a_k(\mathbf{n})\sin[\rho_k(\mathbf{n})]\sin[\psi_k(\mathbf{n})] \\ &\equiv A_{1k}(\mathbf{n})\cos[\psi_k(\mathbf{n})] + A_{2k}(\mathbf{n})\sin[\psi_k(\mathbf{n})]. \end{aligned}$$

Generalized AM and FM functions

Let $f_k(\mathbf{n}) = a_k(\mathbf{n}) \cos[\varphi_k(\mathbf{n})]$.

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\end{aligned}
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&\equiv A_{1k}(\mathbf{n})\cos[\psi_k(\mathbf{n})] + A_{2k}(\mathbf{n})\sin[\psi_k(\mathbf{n})].
\end{aligned}
$$

FM-based: Image Translation

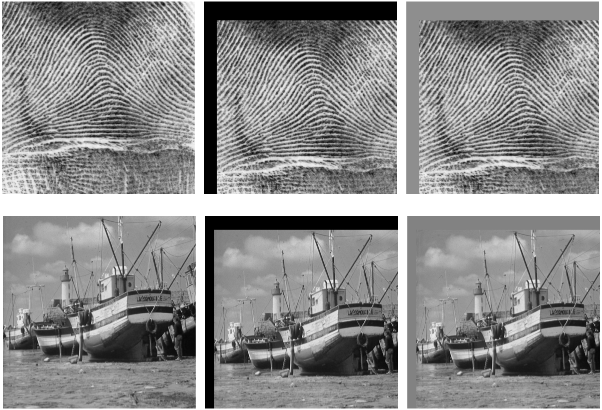


Figure: AM-FM image shift. $\mathbf{u} = (34.70, 50.30)$ and $\mathbf{u} = (24.50, 37.30)$.

Let $L \in \mathbb{R}$ be the scaling factor.

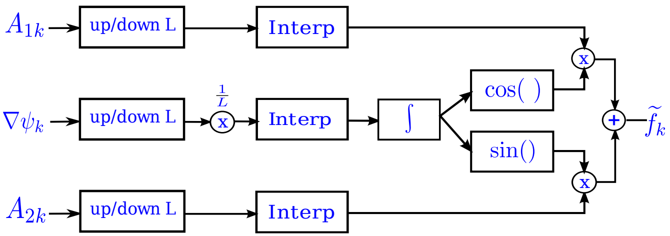


Figure: Block diagram of modulation domain image scaling.

Figure: AM-FM image zoom.

FM-based: Image Scaling

Table: Comparison of the upsampling operation.

	PSNR (dB)		SSIM	
	Bicubic	AM-FM	Bicubic	AM-FM
Boat	33.488	**33.697**	0.765	**0.799**
Barbara	32.091	**32.179**	0.716	**0.728**
Lena	35.018	**35.214**	0.852	**0.862**
Fingerprint	30.362	**30.488**	0.864	**0.869**

Let

$$\mathcal{O}_\alpha = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix} \tag{1}$$

be the rotation matrix by an arbitrary angle α and let $\mathcal{R}_\alpha$ be the rotation operation by an angle α acting on the pixel lattice.

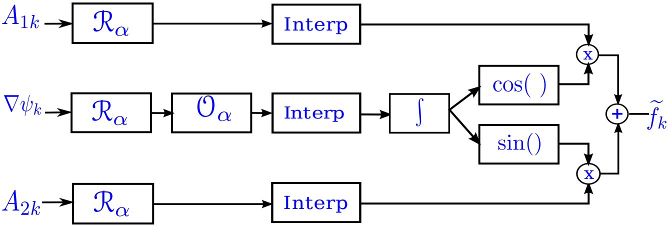


Figure: Block diagram of modulation domain image rotation.

(a) Original.

(b) Spatial 27°.

(c) AM-FM 27°.

(d) Original.

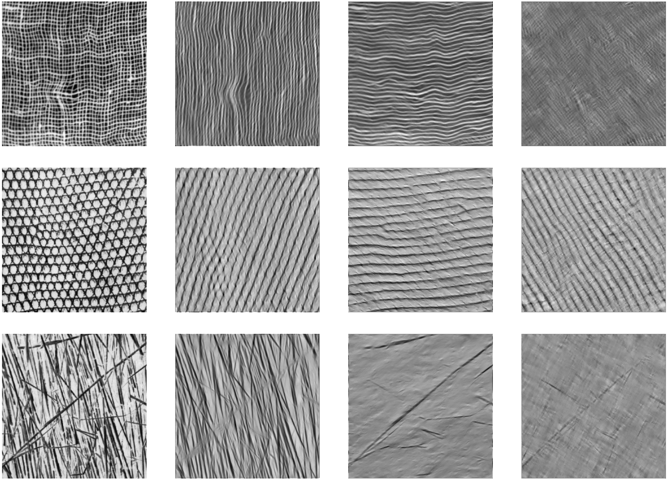
(e) Spatial 45°.

(f) AM-FM 45°.

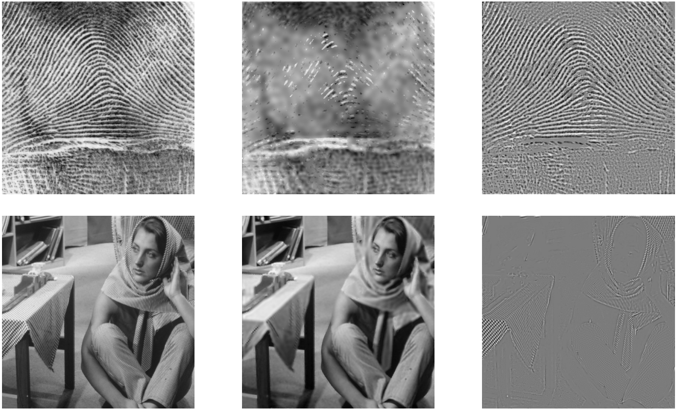
Organization

1. Part I: Theory
   - The AM-FM Image Model
   - The Multi-scale Multi-orientation PR Filterbank
   - The PR AM-FM Transform
2. Part II: Modulation Domain Image Filtering
   - AM-based Image Filtering
   - FM-based Image Filtering
3. Part III: Extensions and Conclusions

Coherent Texture Decomposition



Texture Cartoon Decomposition



Original Contributions

1. New modulation domain image filtering framework with two classes of perceptually motivated image filters.
2. A true multi-orientation multi-scale PR filterbank.
3. An artifact-free AM-FM demodulation algorithm.
4. A perfect reconstruction FM algorithm.
5. Extensions of the xAMFM for image decomposition problems.

Future Research

- New field of perceptually motivated image filtering.
- Foundation for general/sophisticated filters.
- Effect of noise.
- Image and video quality assessment.
- The challenging data driven decomposition.

Conclusion

- Developed the PR AM-FM transform (xAMFM).
- Designed perceptually motivated modulation domain filters.
- Used the xAMFM in image processing applications.

Thank you!

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